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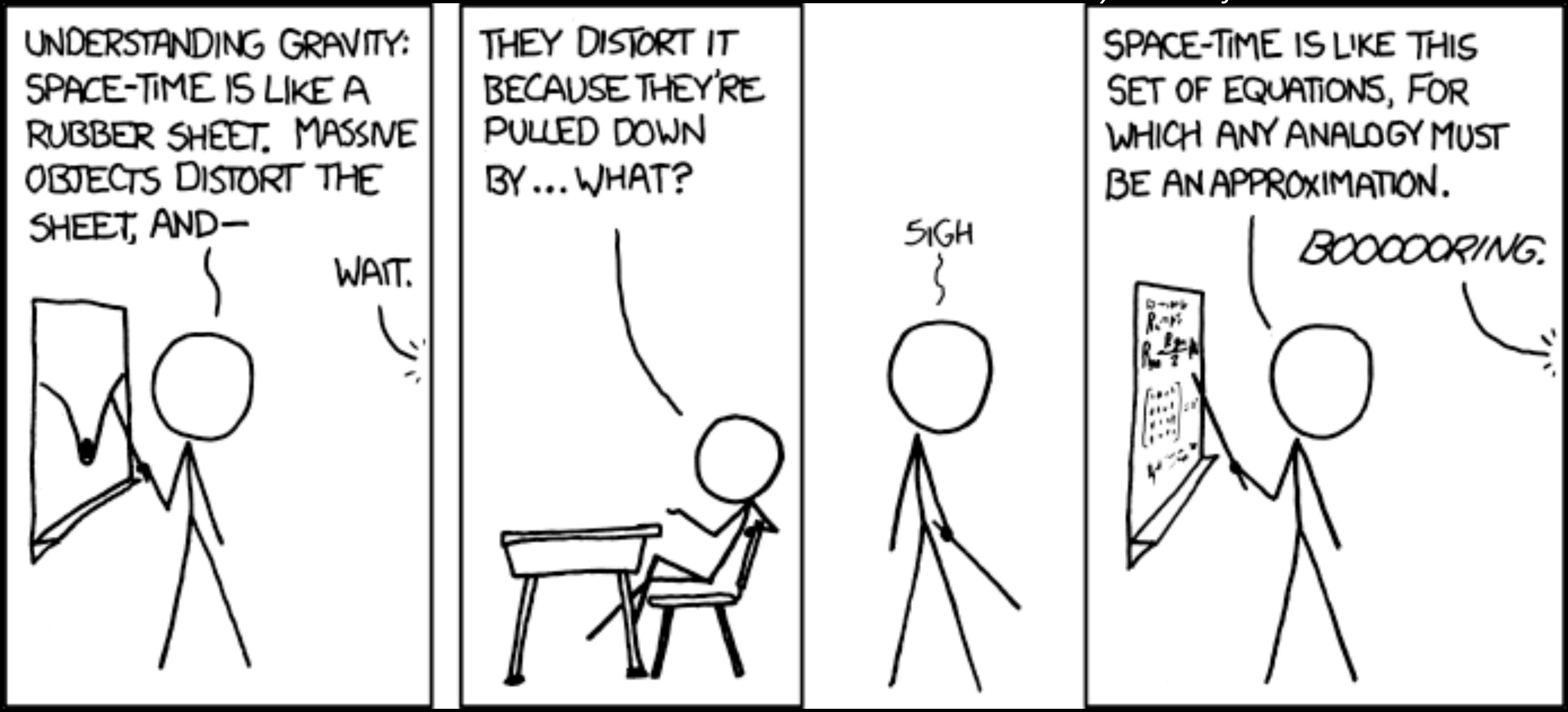
The Physics of LIGO, Virgo, & KAGRA

GW science over the next decade

Patrick Sutton
Cardiff University

my second-favourite xkcd.com comic



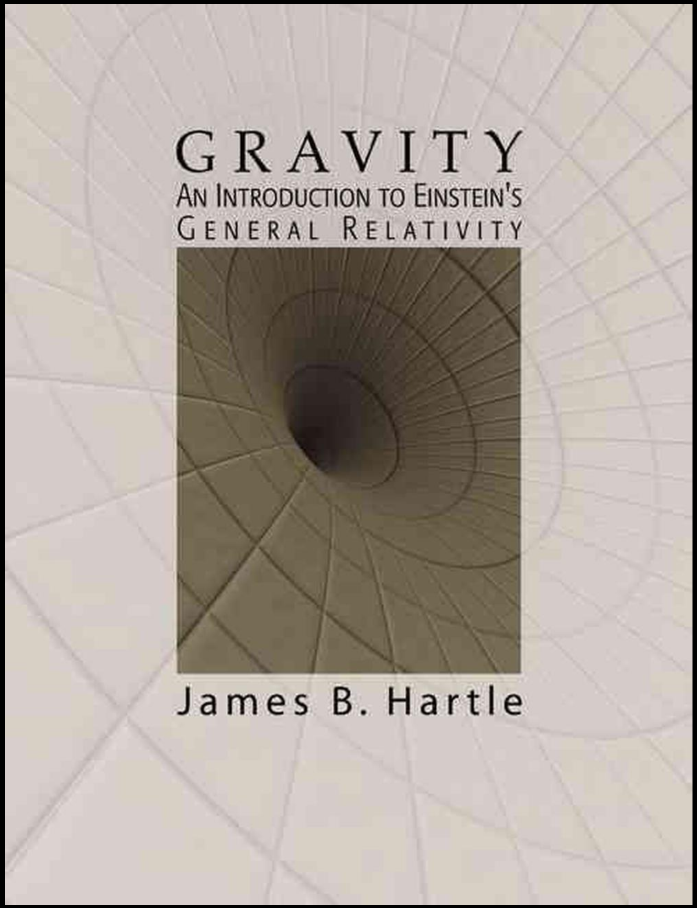
Content of the next few lectures

- **Patrick, Lecture 1 (now):** Basic principles behind gravitational-wave astronomy
  - GW emission & the quadrupole approximation
  - Main source types and how we search for them
  - Detector networks: detection confidence & sky localization

- **Erik, Lectures 1 & 2 (today, tomorrow):** Science results to date from LIGO-Virgo

- **Patrick, Lecture 2 (Fri):** Future science
  - LIGO, Virgo, KAGRA & LIGO-India
  - Cosmic Explorer & the Einstein Telescope

GWs in Linearized Gravity



GRAVITY
AN INTRODUCTION TO EINSTEIN'S
GENERAL RELATIVITY

James B. Hartle

New Journal of Physics
The open–access journal for physics

The basics of gravitational wave theory

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Abstract. Einstein's special theory of relativity revolutionized physics by teaching us that space and time are not separate entities, but join as 'spacetime'. His general theory of relativity further taught us that spacetime is not just a stage on which dynamics takes place, but is a participant: the field equation of general relativity connects matter dynamics to the curvature of spacetime. Curvature is responsible for gravity, carrying us beyond the Newtonian conception of gravity that had been in place for the previous two and a half centuries. Much research in gravitation since then has explored and clarified the consequences of this revolution; the notion of dynamical spacetime is now firmly established in the toolkit of modern physics. Indeed, this notion is so well established that we may now contemplate using spacetime as a *tool* for other sciences. One aspect of dynamical spacetime—its radiative character, 'gravitational radiation'—will inaugurate entirely new techniques for observing violent astrophysical processes. Over the next 100 years, much of this subject's excitement will come from learning how to exploit spacetime as a tool for astronomy. This paper is intended as a tutorial in the basics of gravitational radiation physics.

New Journal of Physics **7** (2005) 204
1367-2630/05/010204+51$30.00
PII: S1367-2630(05)92710-9

GWs in Linearized Gravity

- Concrete treatment based on linearized perturbations around a fixed background metric in general relativity.
- Perturbation obeys a wave equation. E.g., in flat space:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar{h}_{ij} = -\frac{16\pi G}{c^4} T_{ij}$$

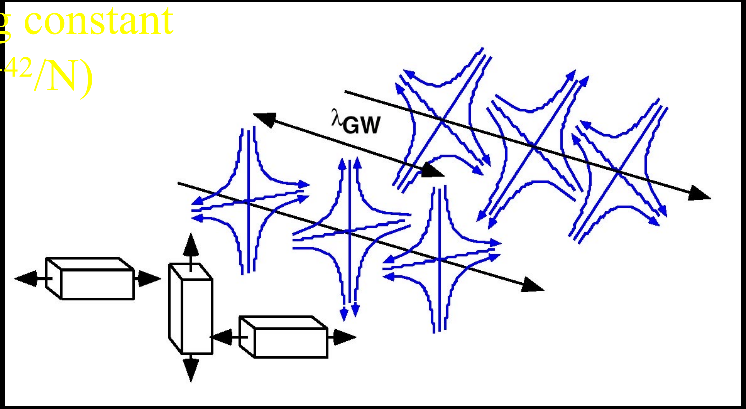
source stress tensor (density of mass & energy)

wave operator

perturbation (trace-reversed)

coupling constant (10^{-42}/N)

- Solution: 2 radiative polarization states ("+" and "x") rotated by 45º
  - Quadrupolar (tidal) fluctuations in geometry
  - Travel at speed of light (c)



Quadrupole Approximation

- Valid for slow moving sources (v << c) with weak gravity ($\Phi/c^2 << 1$).

$$h^{ij}(t) = P_{\mathrm{TT}} \left[\frac{2}{r} \frac{G}{c^4} \frac{d^2}{dt^2} \int_{\mathrm{source}} d^3\vec{x}\; \mu(t-r,\vec{x})\, x^i x^j \right]$$

transverse-traceless projection

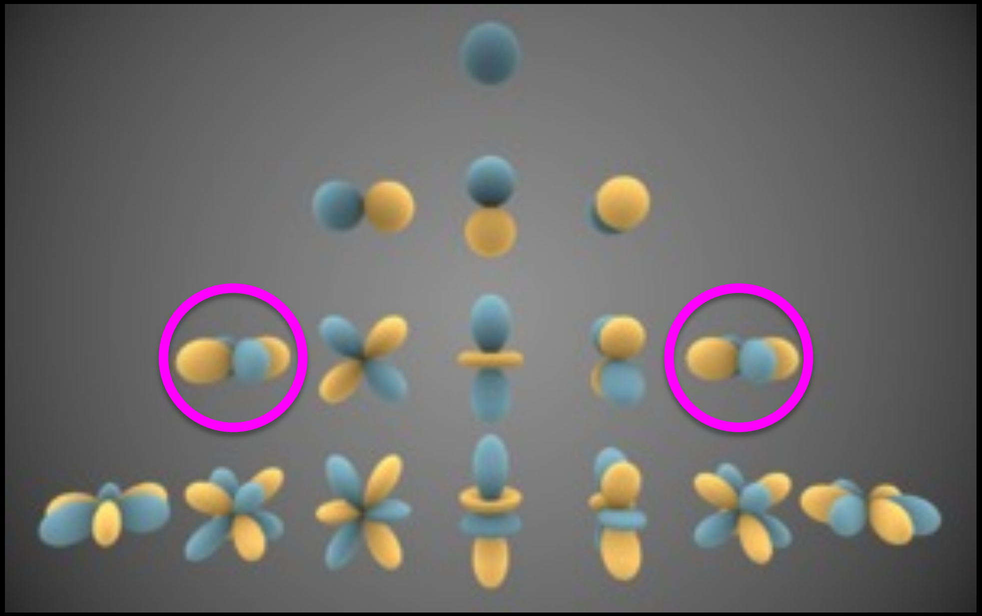
Newtonian mass-density of source

- Sufficient for estimating what we can / can’t detect.
  - E.g., supernova simulations typically use this approximation to estimate GW emission.

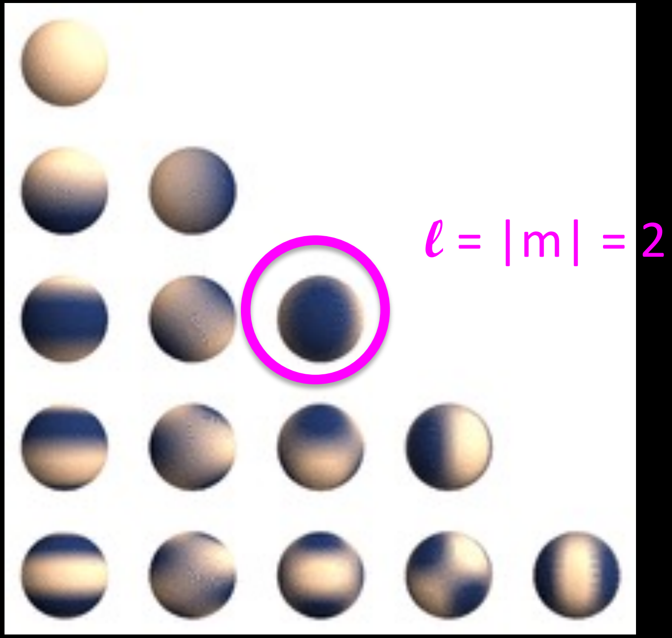
- Post-Newtonian (PN) expansion goes beyond v << c approximation, numerical relativity (NR) goes beyond weak field (Alessandra’s lectures); these are needed for accurate estimation of most binary source parameters.

Quadrupole Approximation

- Spherical harmonics: only the ℓ = |m| = 2 emission mode is non-zero.



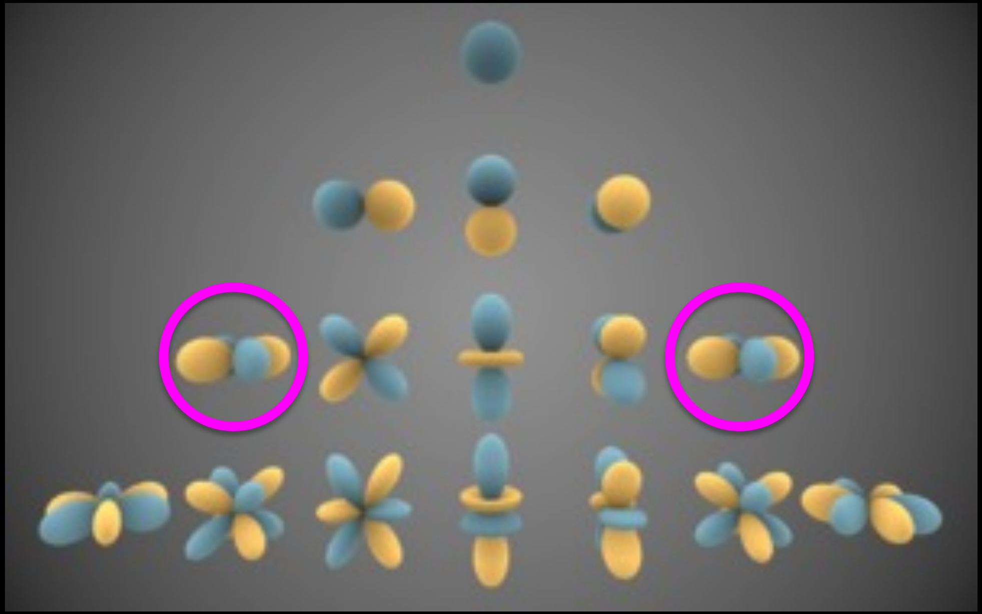
credit: Wikipedia (Inigo.quilez)



credit: Wikipedia (Cyp)

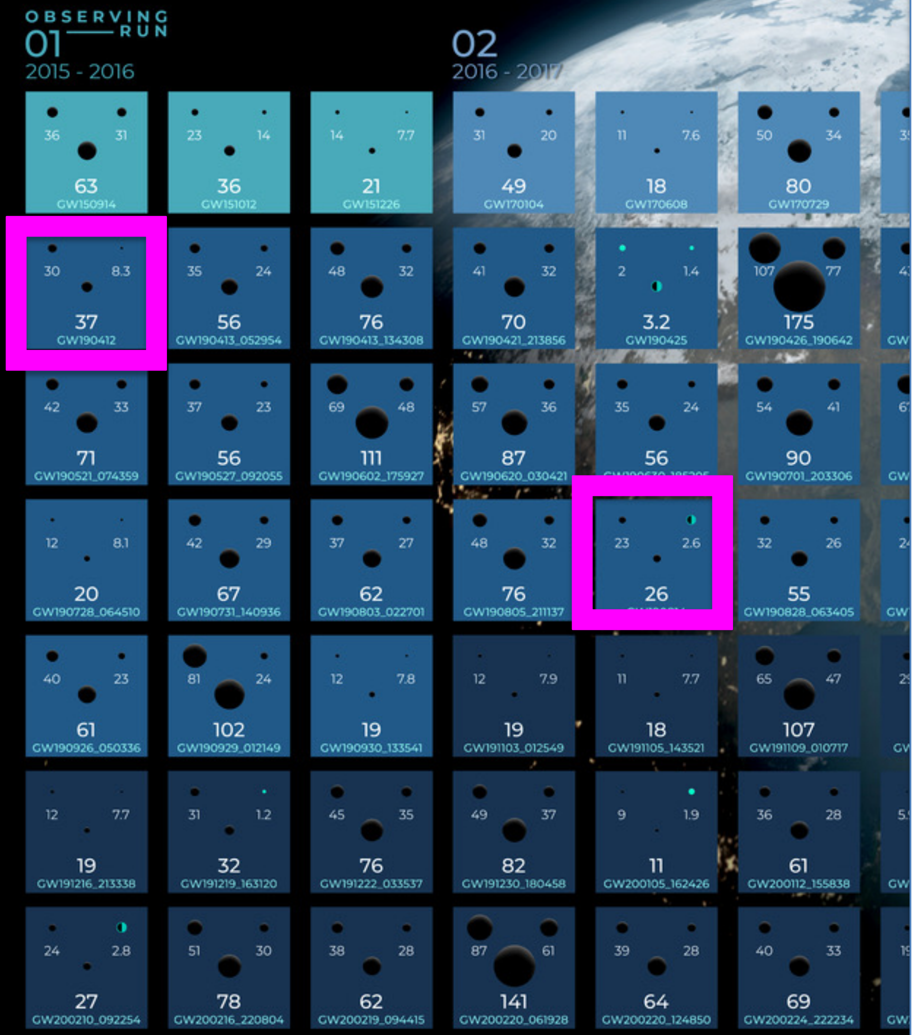
Quadrupole Approximation

- Spherical harmonics: only the $\ell = |m| = 2$ emission mode is non-zero.



credit: Wikipedia (Inigo.quilez)

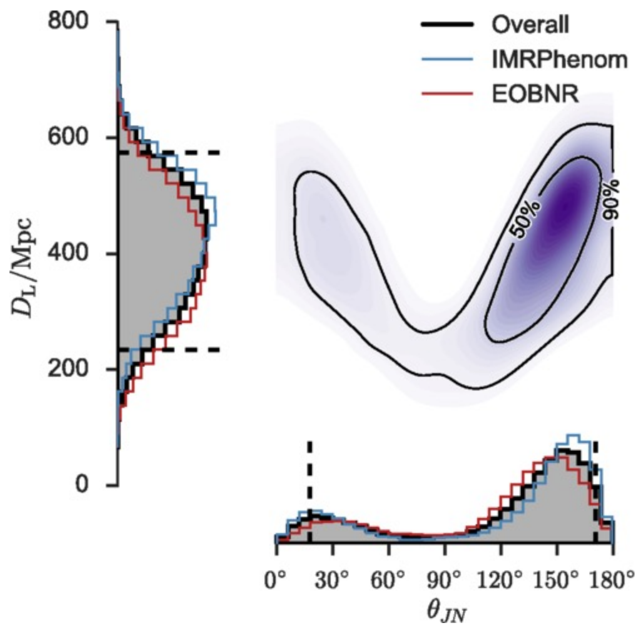
- Higher-mode (ℓ>2) emission is only detectable from a small fraction of sources.



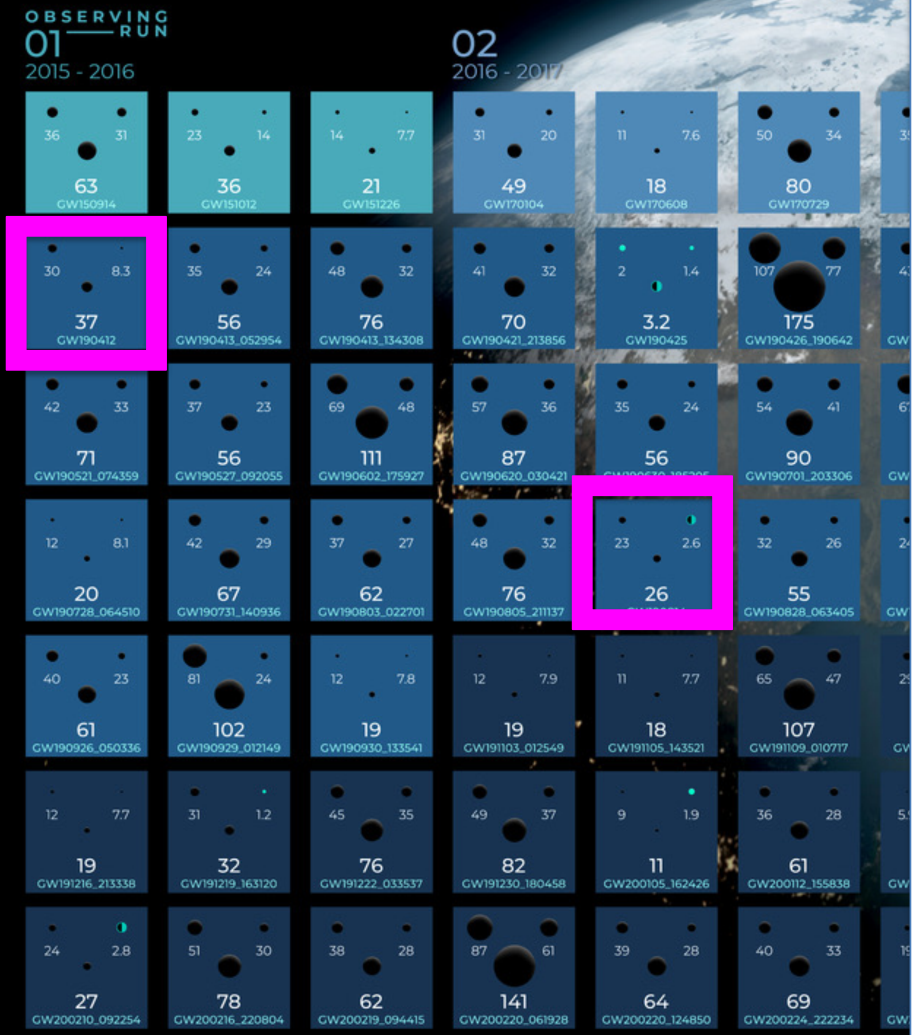
May allow to break the inclination-distance degeneracy

- C. Cutler & E. Flanagan, PRD49, 2658 (1994);
  S. Usman et al., ApJ, 877 82 (2019)
- vital for H_0 measurements

GW150914:
distance vs.
inclination



Abbott et al. PRL116 241102 (2016)



May allow to break the inclination-distance degeneracy

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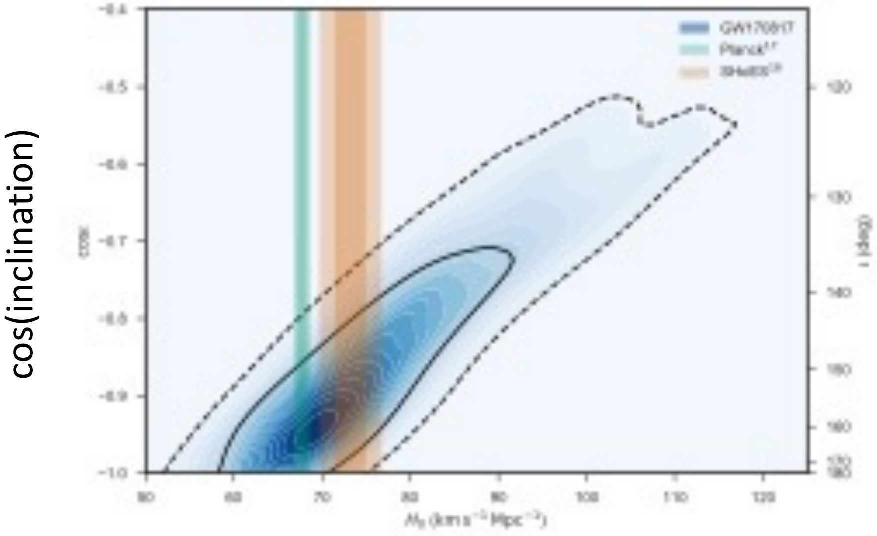
GW170817:



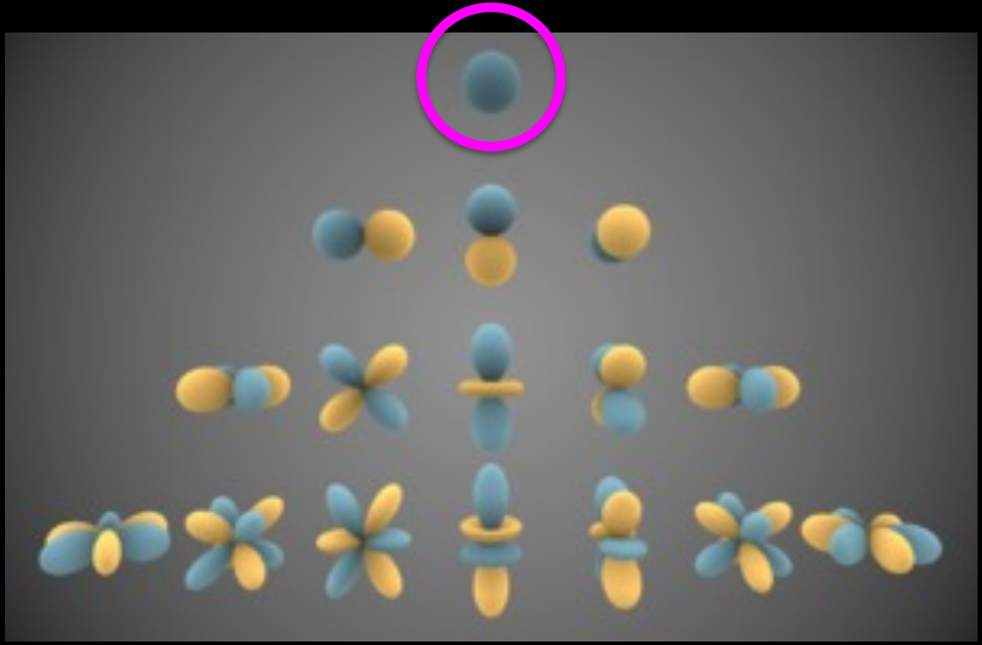
cos(inclination)

Hubble constant

B. P. Abbott et al., Nature 551, 85 (2017)

Quadrupole Approximation

- Spherical harmonics: only the $\ell = |m| = 2$ emission mode is non-zero.



credit: Wikipedia (Inigo.quilez)

- Purely spherically symmetric motion = zero GW emission in full GR too (*"Birkhoff theorem"*).
  - Bad news for, e.g., supernovae.

Detectable Sources

Source

- Rotating quadrupole:

mass frequency size

$$h \sim \frac{\epsilon\, M\, \Omega^2\, R^2}{r} \sim \frac{\epsilon\, M^2}{Rr}$$

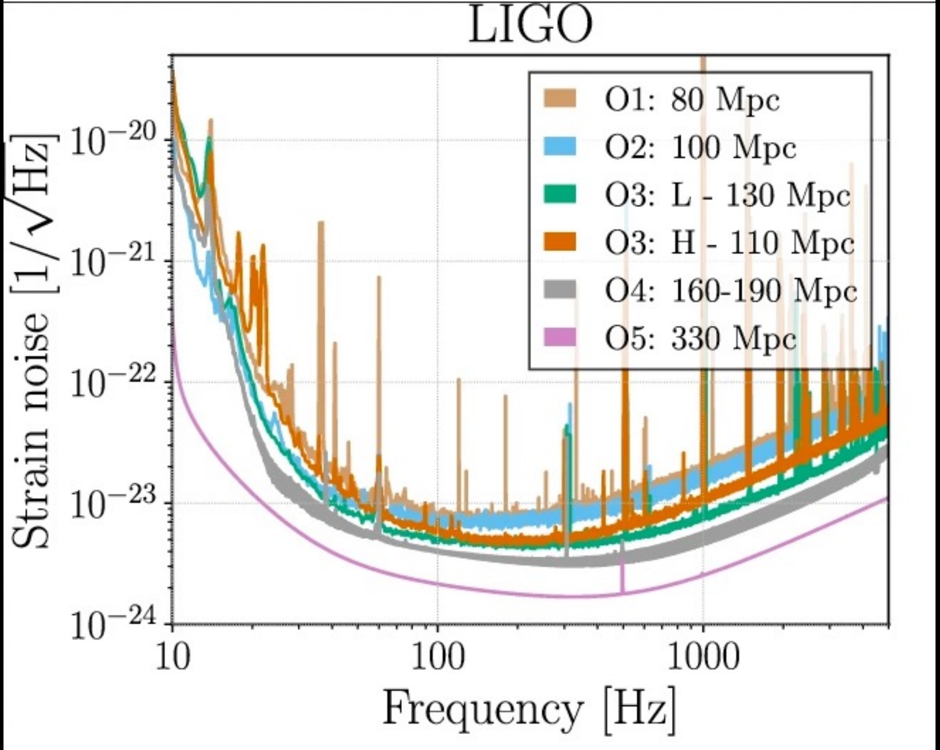
non-sphericity distance Kepler

- LIGO/Virgo sensitive band ~30-3000 Hz implies maximum size of source:

$$R \lesssim \frac{c}{f} = 100\,\mathrm{km} - 10,000\,\mathrm{km}$$

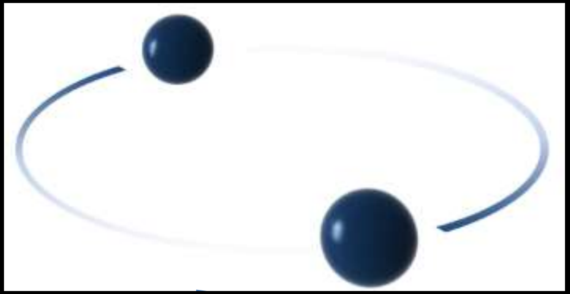
Detector

Abbott et al. 1304.0670



Small and Dark

- High mass, small radius: sources are very high density
  - Tend to see the cores of objects (e.g. supernova inner cores), rather than the surface as in EM astronomy
- Prime sources are compact objects: NSs, BHs, and proto-NS cores
  - WD binaries are out of band (too low frequency)
  - GW170817 @ 40 Mpc: $h < 3\text{x}10^{-22}$ at Earth



- LIGO/Virgo see the corpses of high-mass stars.


I SEE DEAD PEOPLE
STARS

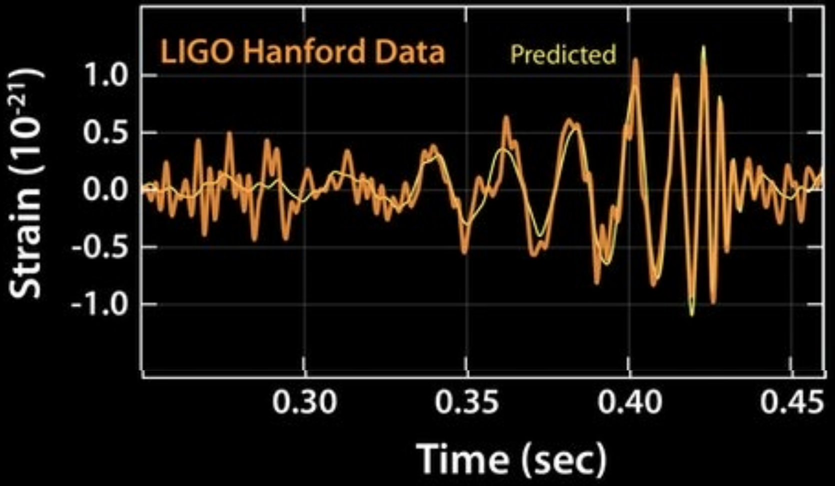
MAIN GW SOURCE TYPES

Compact Binaries Coalescences (CBCs)

- Inspiral, merger, and ringdown of binaries of black holes and/or neutron stars
  - signals modelled accurately by PN+NR (Alessandra's lectures).
  - matched filter on short stretches of data [O(10^2) sec].

GW150914 – the first BH-BH merger

Abbott et al. PRL **116** 061102



Approximate Masses:
$36\ M_\odot + 29\ M_\odot$
Energy emitted:
$3\ M_\odot\ c^2$
Peak luminosity:
$200\ M_\odot\ c^2/s$
(3.6×10^{56} erg/s)

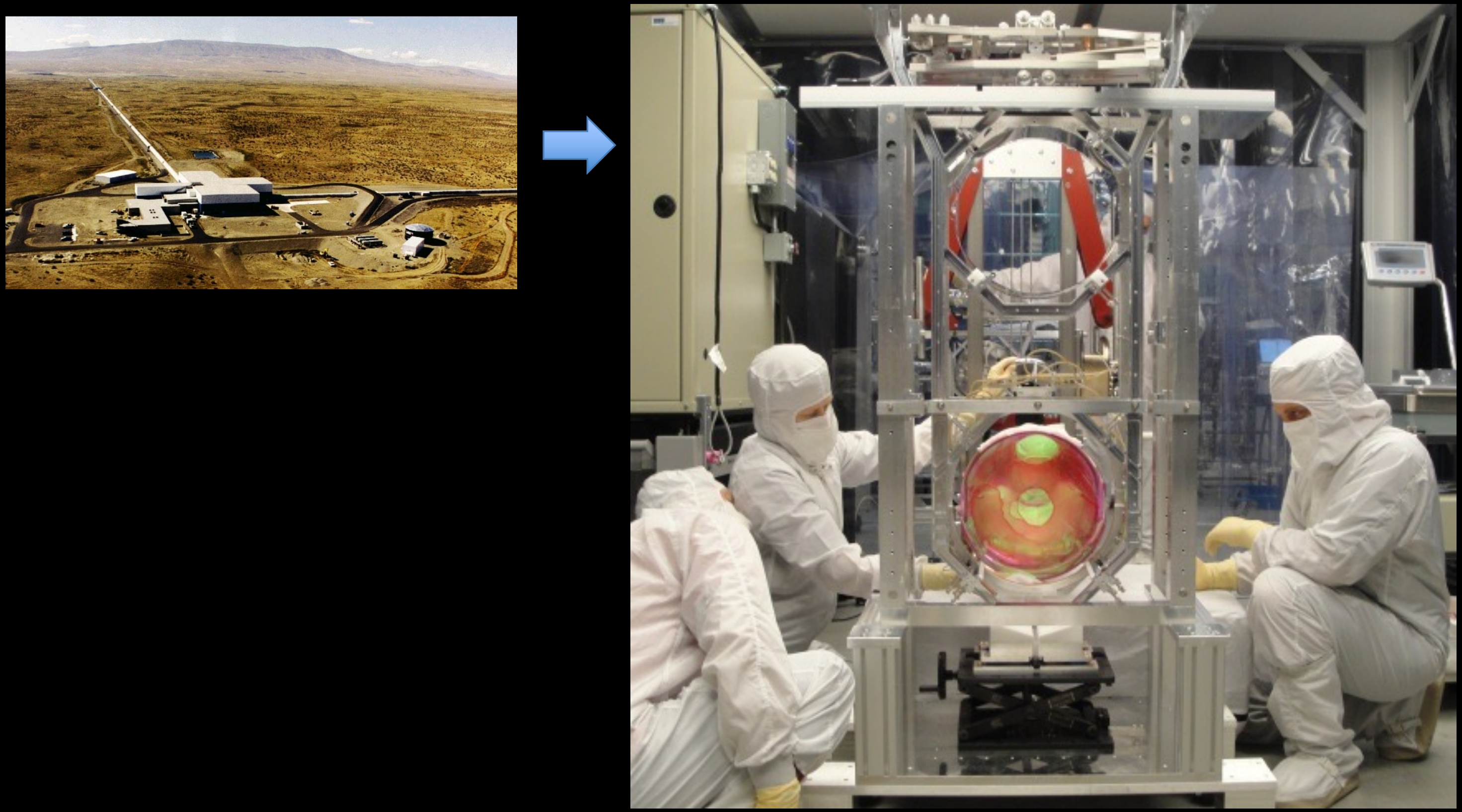
The Challenge:

GW150914 stretched LIGO's arms by one part in 10^{21}*…*

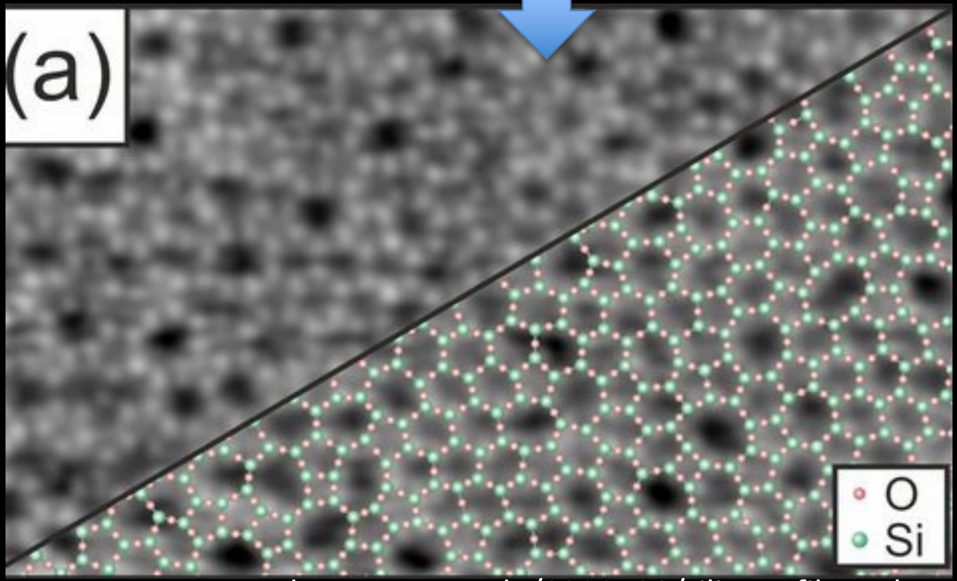
LIGO Hanford Observatory

LIGO Hanford Observatory

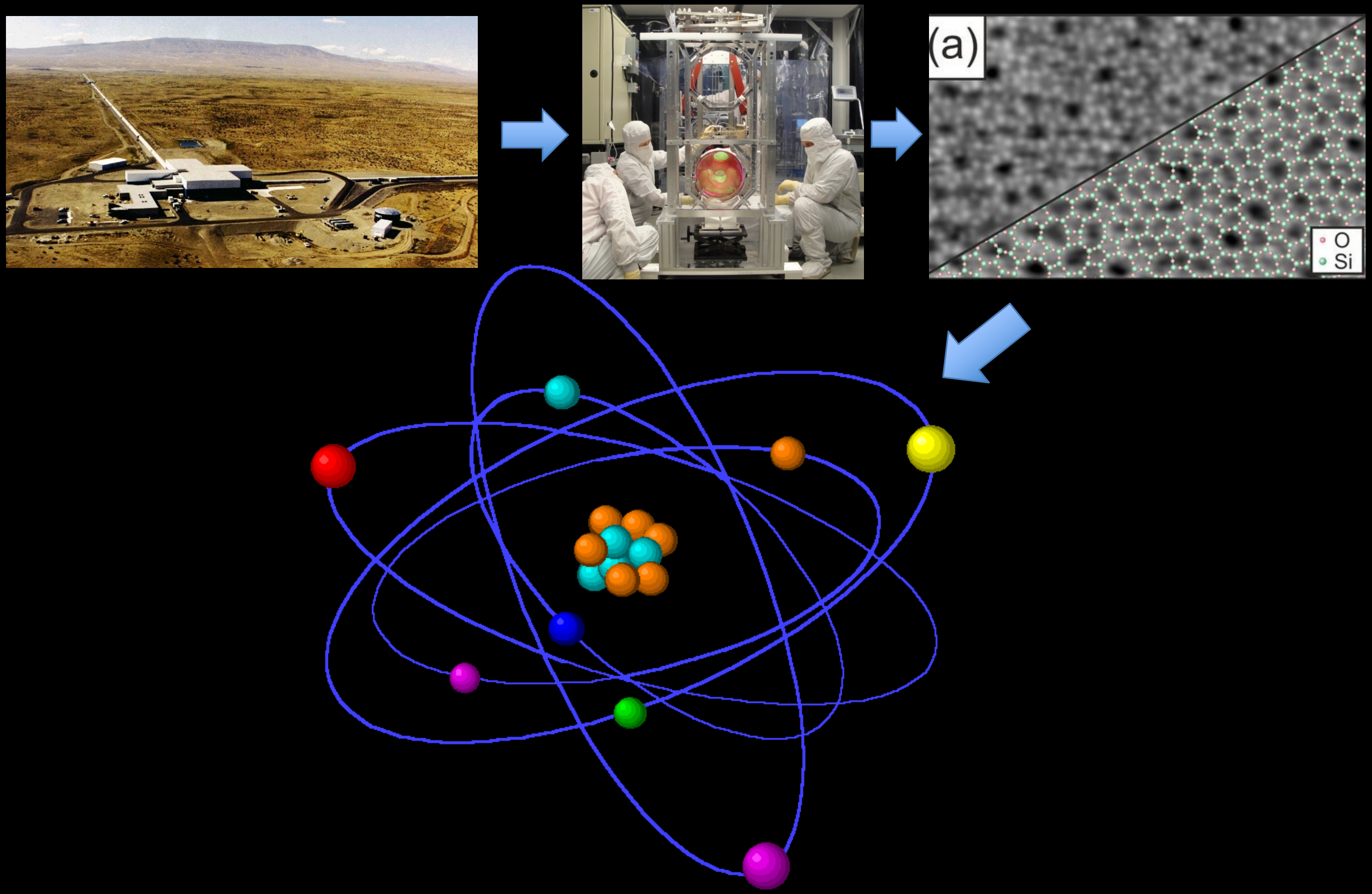
zoom factor: 10,000



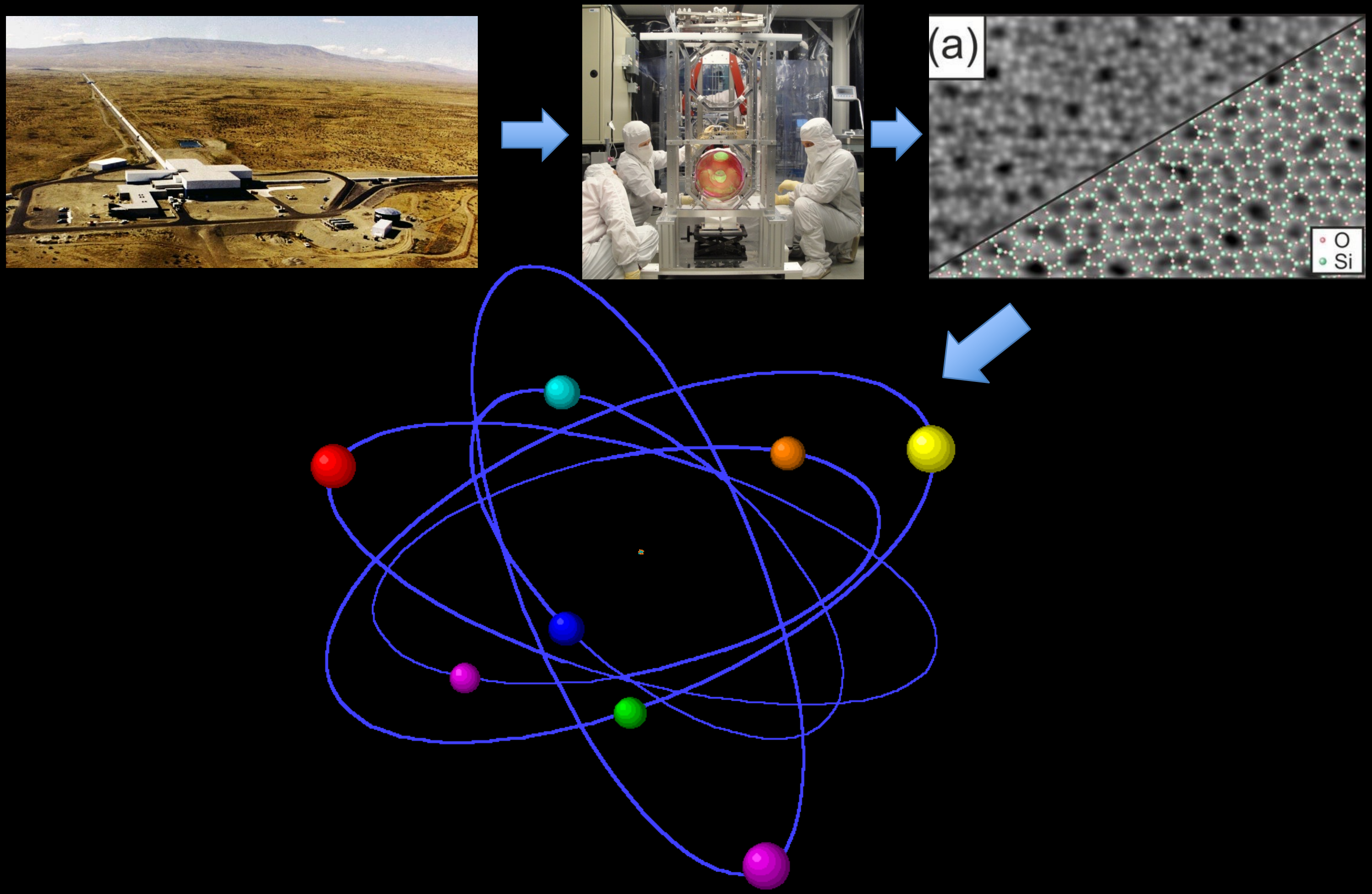
zoom factor: 10,000 x 100,000,000



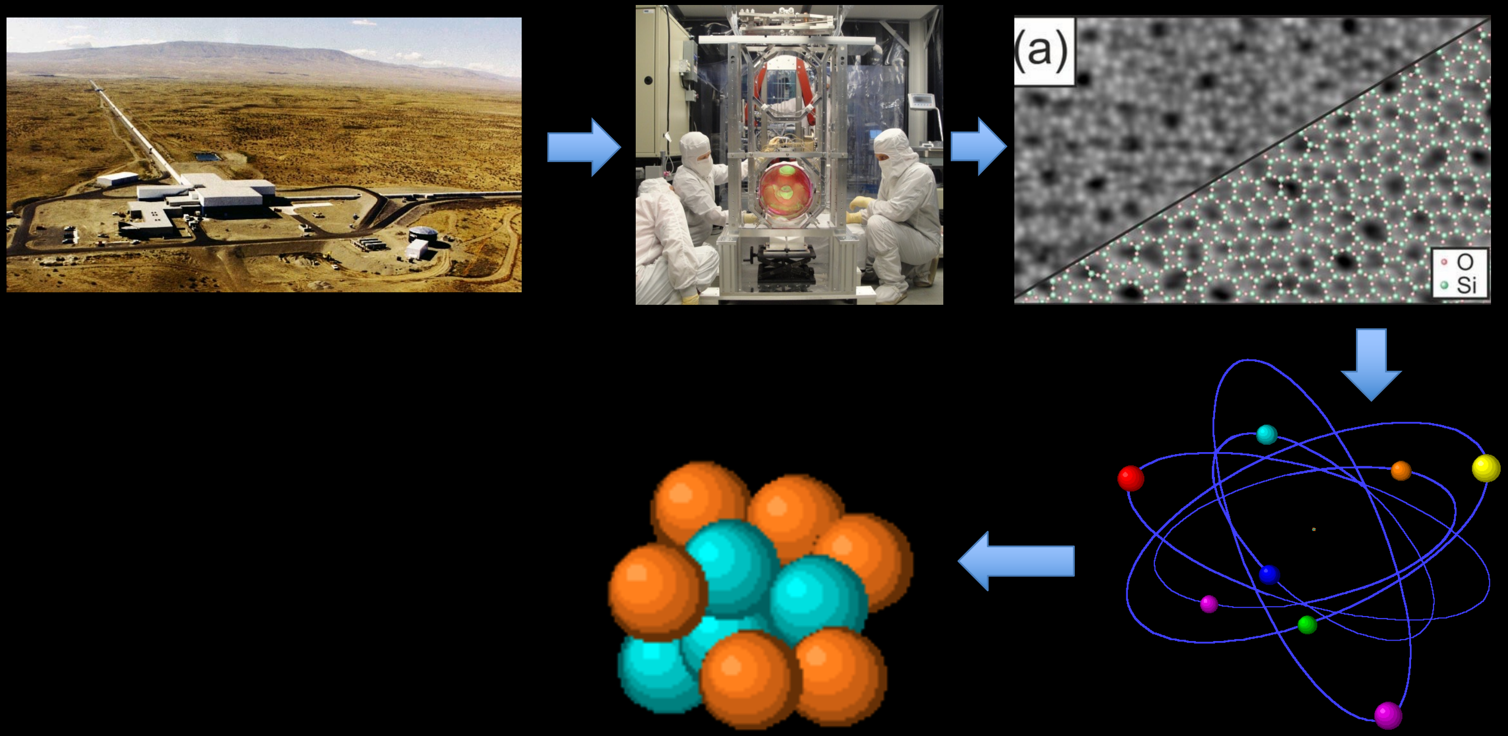
M. Heyde, www.mpg.de/8239438/silicate-films-glass zoom factor: 10,000 x 100,000,000 x 100



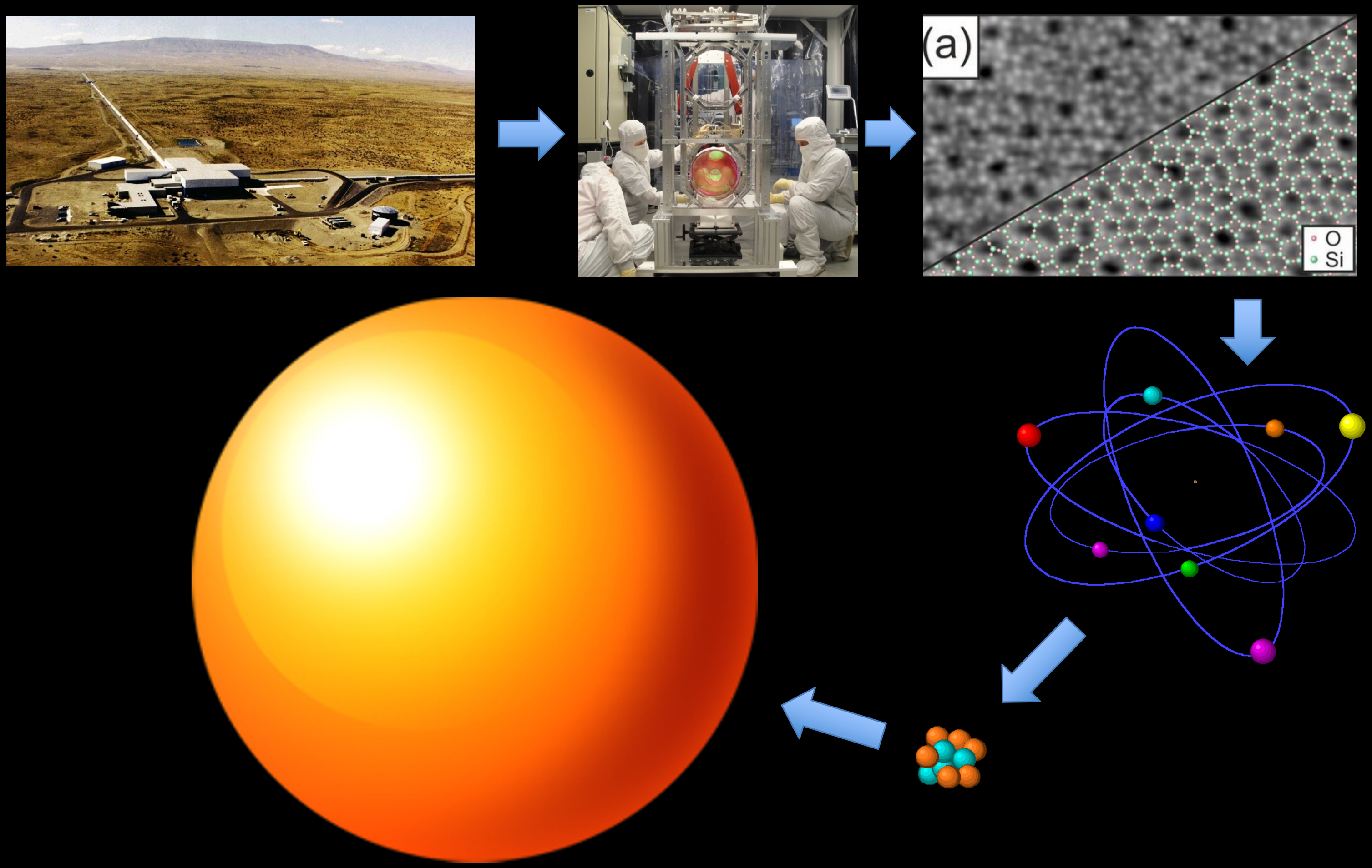
zoom factor: 10,000 x 100,000,000 x 100



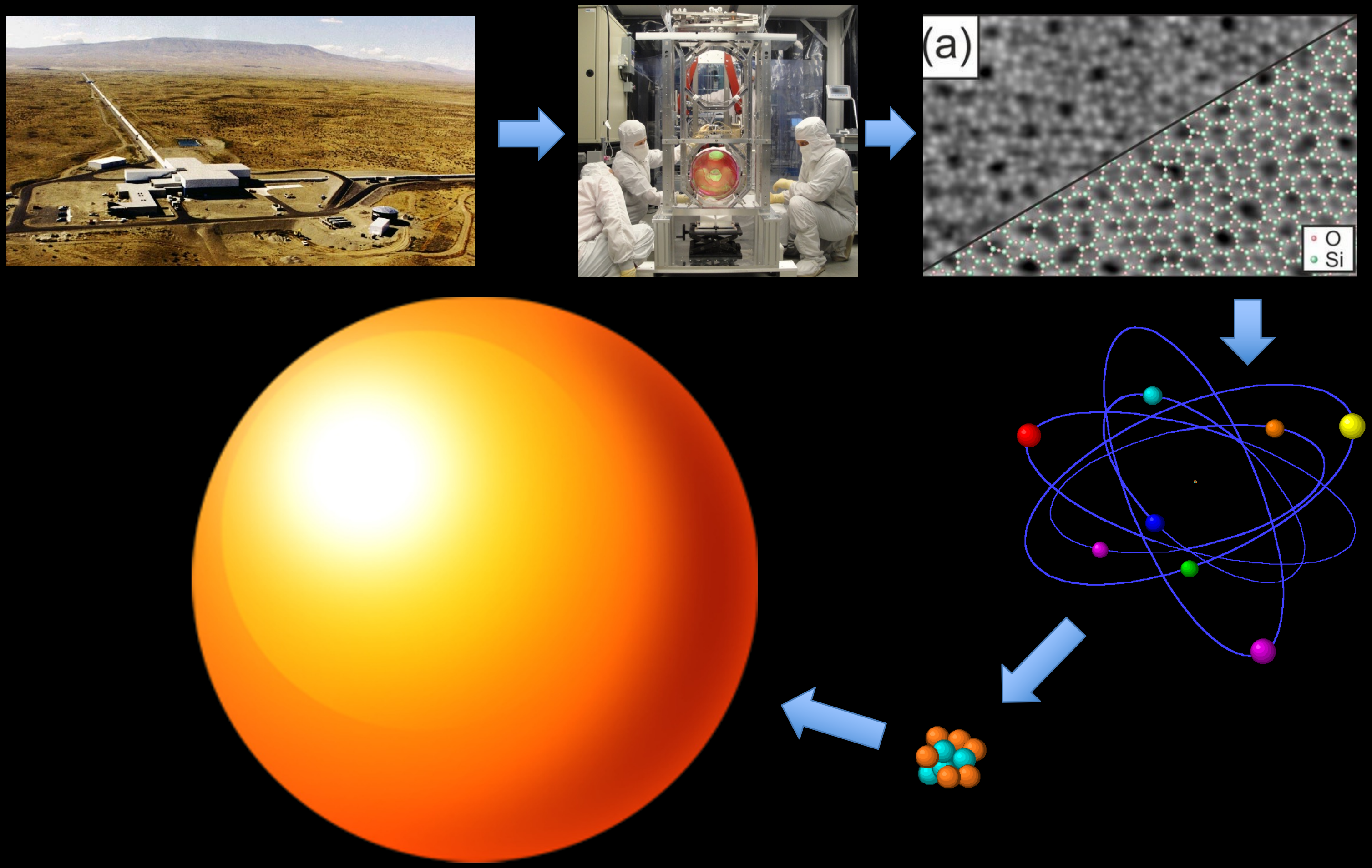
zoom factor: 10,000 x 100,000,000 x 100 x 10,000



zoom factor: 10,000 x 100,000,000 x 100 x 10,000 x 10

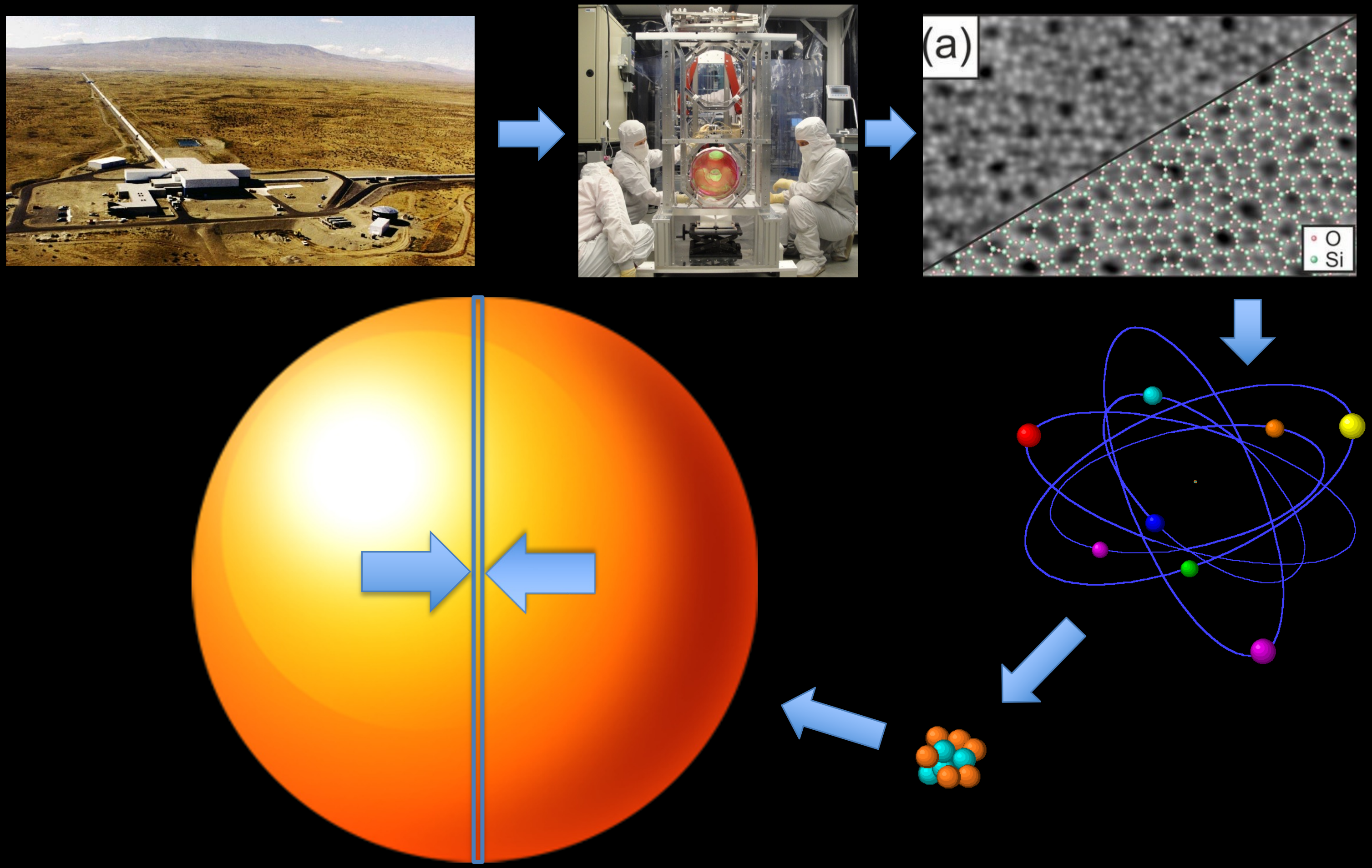


zoom factor: 10,000 x 100,000,000 x 100 x 10,000 x 10 = 10^{19}

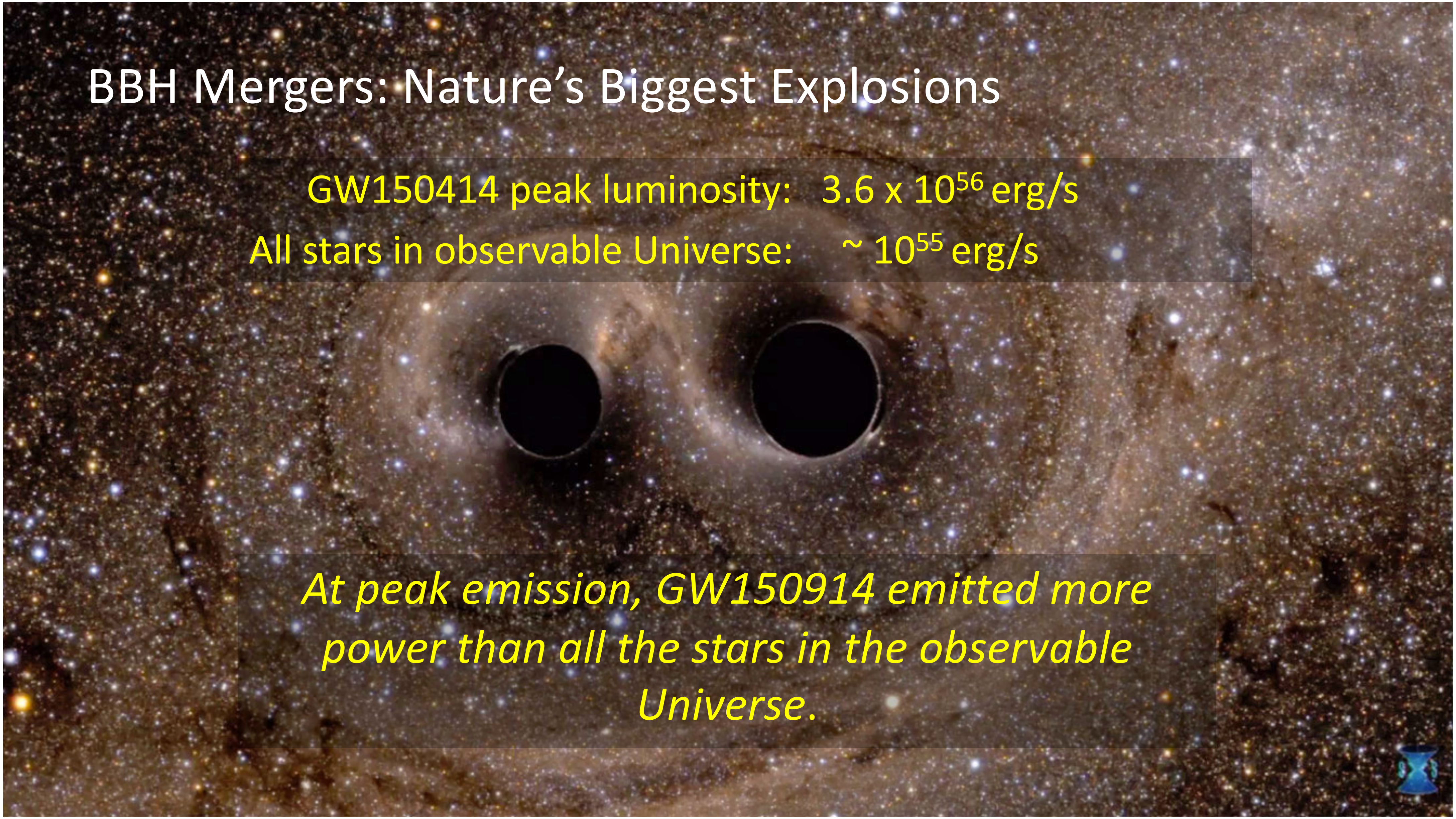


zoom factor: 10,000 x 100,000,000 x 100 x 10,000 x 10 x 100

= 10^{21}



BBH Mergers: Nature's Biggest Explosions



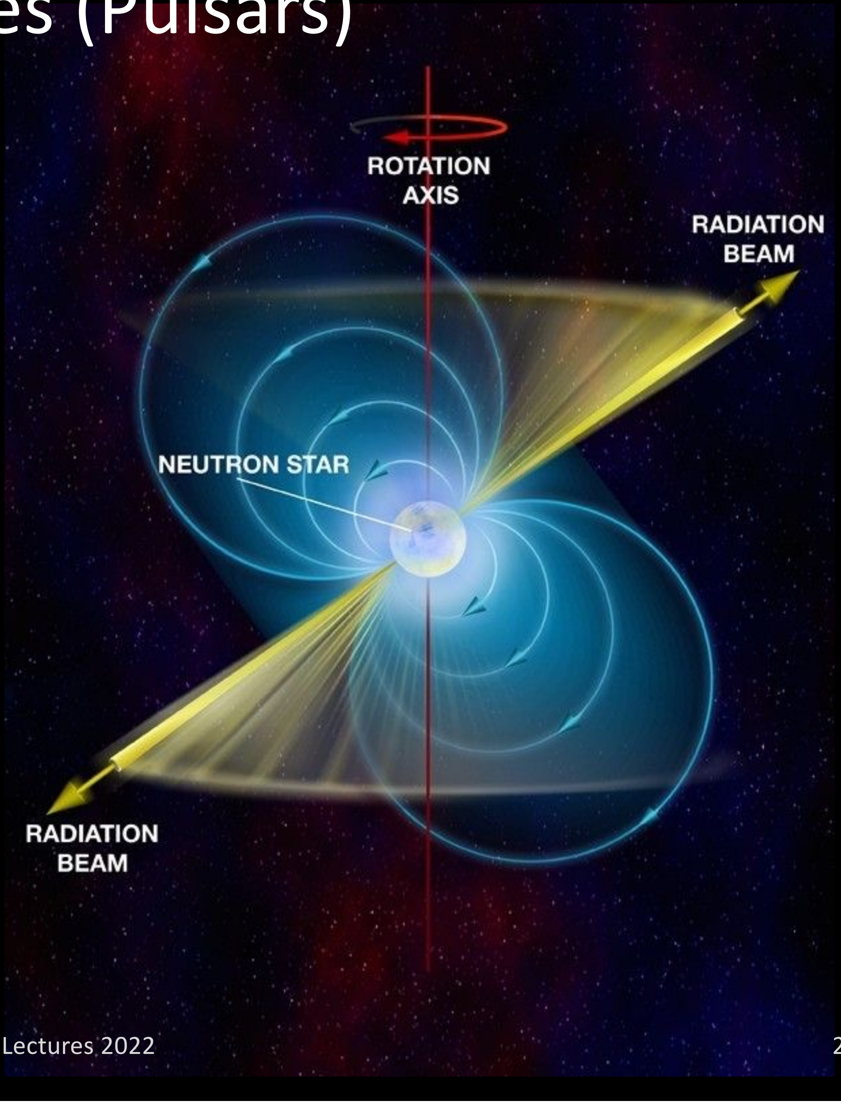
GW150414 peak luminosity: 3.6×10^{56} erg/s

All stars in observable Universe: $\sim 10^{55}$ erg/s

At peak emission, GW150914 emitted more power than all the stars in the observable Universe.

Continuous Waves (Pulsars)

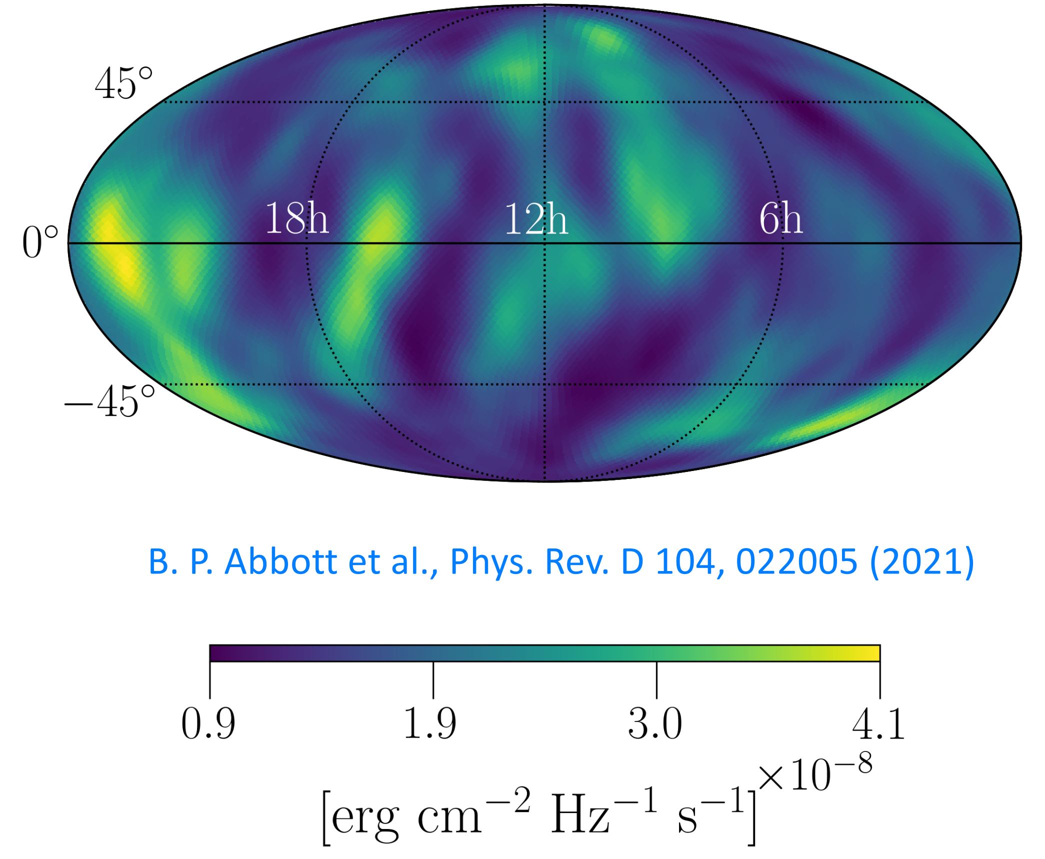
- Rotating non-axisymmetric neutron stars
  - signals modelled accurately as spinning quadrupole with assumed frequency & spindown rate (sine wave with slowly decreasing frequency).
  - matched filter on long stretches of data [O(1) year ideally; O(10^3) sec in practice].



Gravitational-Wave Bursts

- Unmodelled transient signals:
  - from unknown/poorly modelled transient sources, eg supernovae, accretion disk instabilities, neutron-star transients.
  - excess power correlated between detectors on millisec-sec timescales

Stochastic Gravitational-Wave Background



B. P. Abbott et al., Phys. Rev. D 104, 022005 (2021)

- Random signal due to superposition of many weak unresolved binary/other sources (astrophysical) or from early universe (cosmological)
  - cross-correlate pairs of detectors on short (<1 sec) timescales.

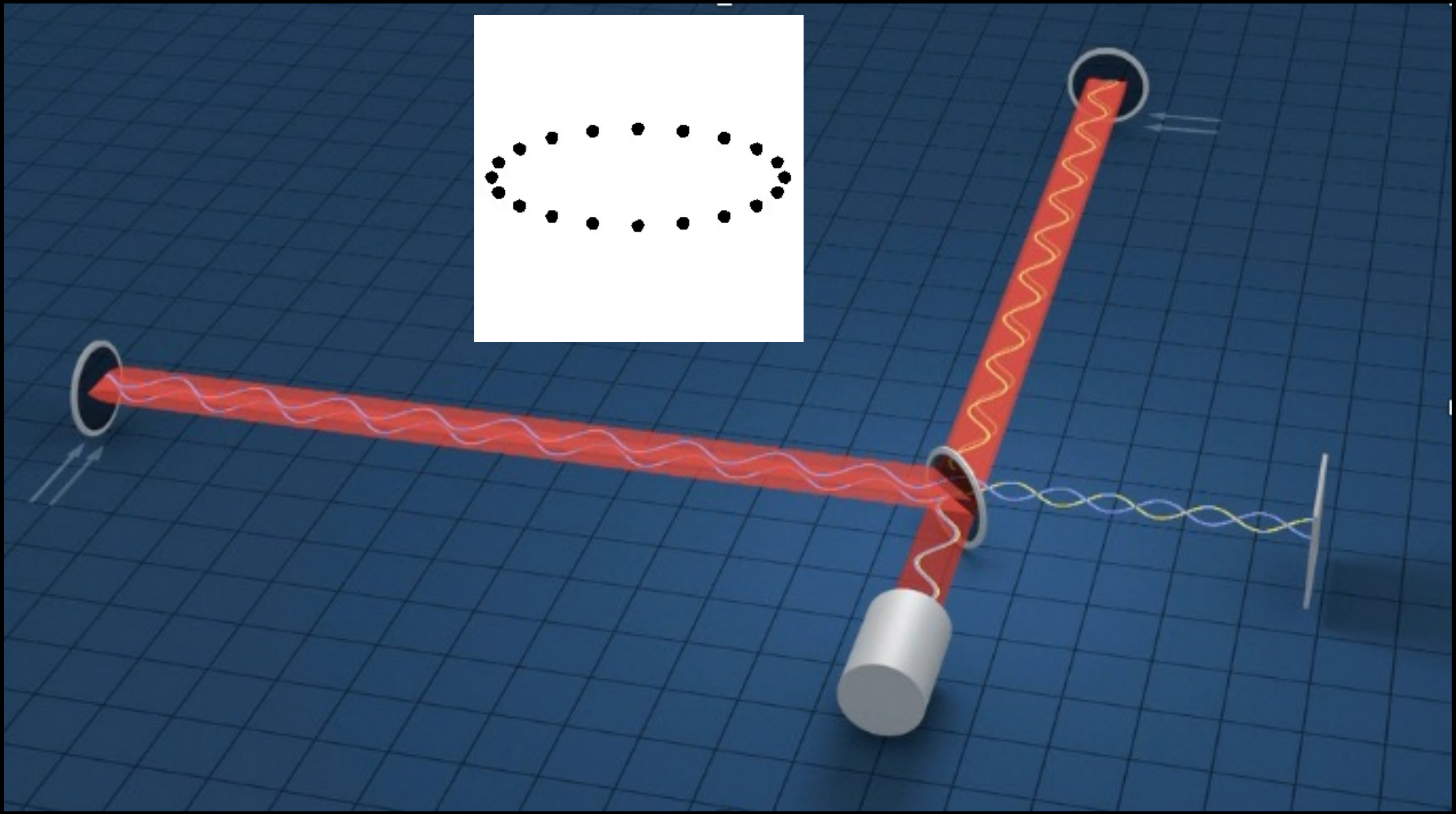
GW DETECTOR NETWORKS

Astrophysics with GWs vs. EM

ElectroMagnetic waves	**Gravitational Waves**
Accelerating charge	Accelerating aspherical mass
Wavelength small compared to sources → images	Wavelength large compared to sources → no spatial resolution
Absorbed, scattered, dispersed by matter	Very small interaction; matter is transparent
10 MHz and up	10 kHz and down

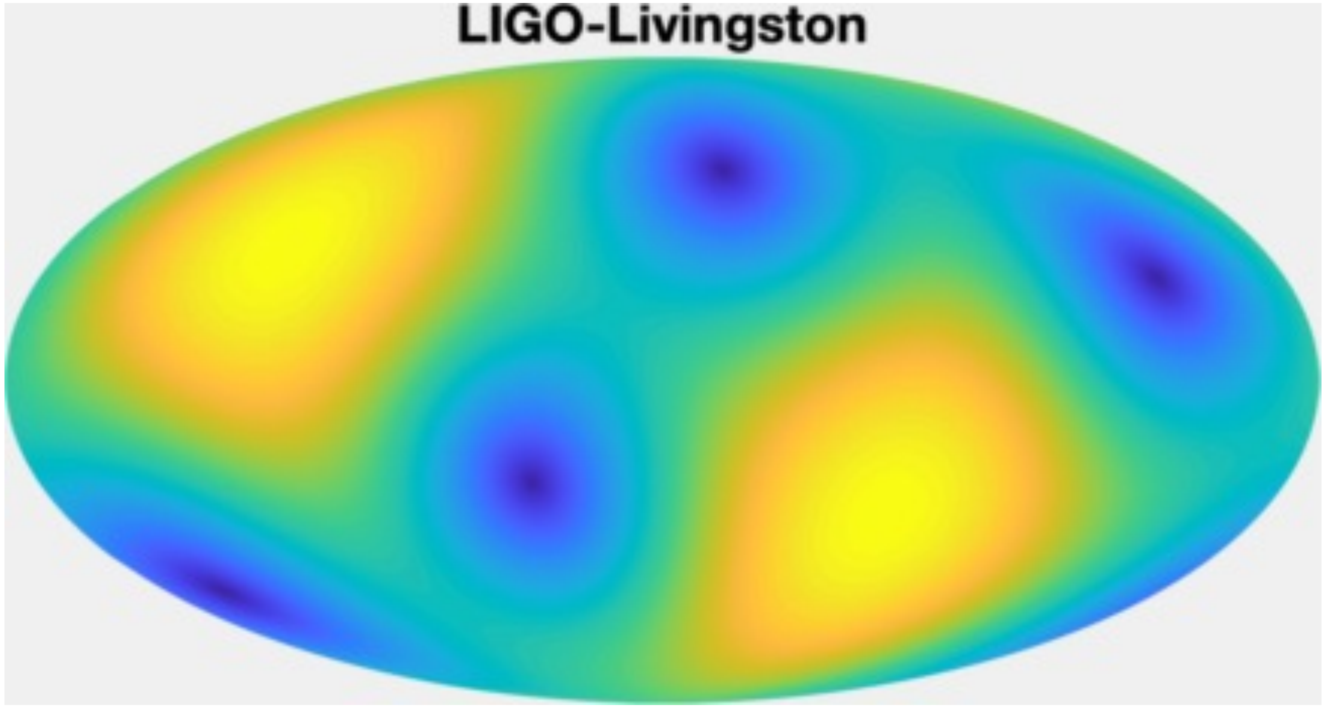
- *GW detectors are all-sky, low bandwidth.*
  - low latency & archival searches: easy.
  - source localization: hard.
- *Complementary to EM observatories*

GW Detectors: Interferometers



Single Detector Case

A single detector observes most of the sky …



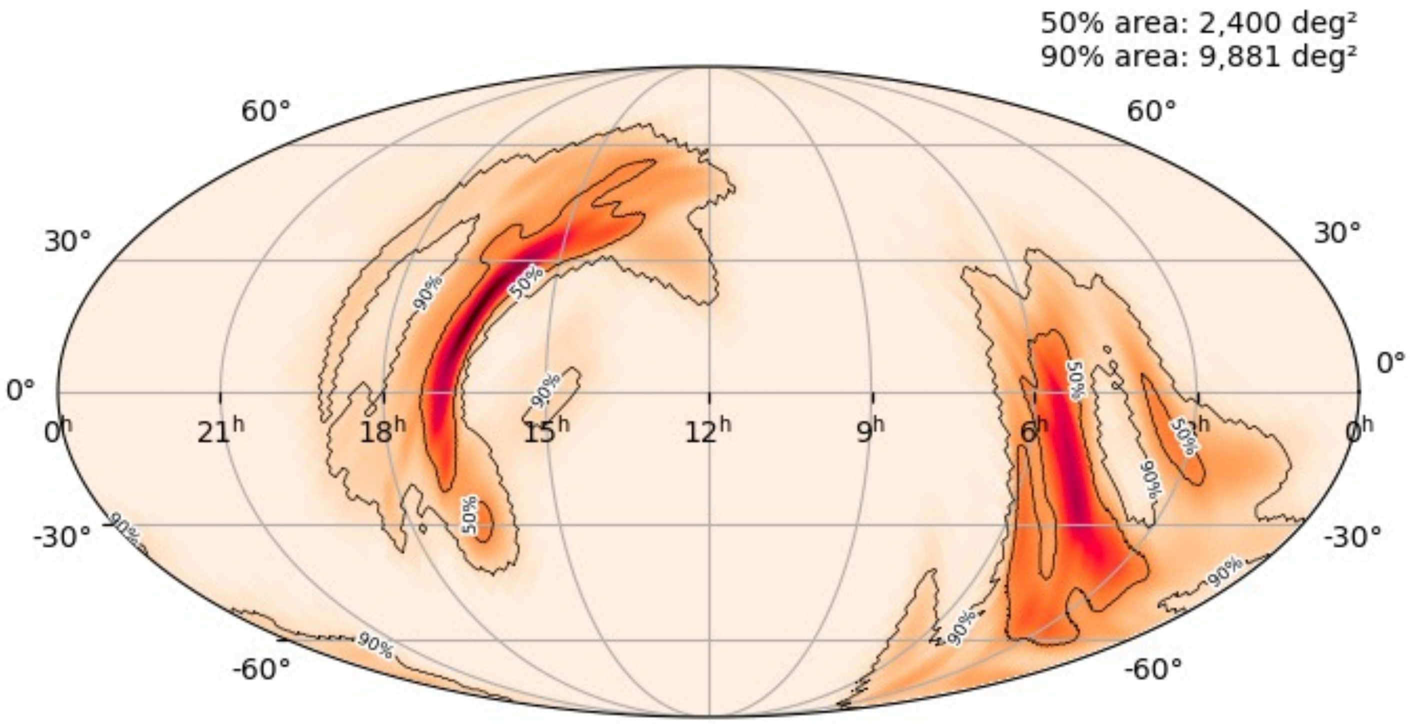
$$F_{+} = -\tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi$$
$$F_{\times} = +\tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi - \cos\theta\sin 2\phi\cos 2\psi$$

W. Anderson et al. PRD63 042003 (2001)

LIGO Livingston RMS response over the sky
(> 0.5 maximum over 65% of the sky)

Single Detector Case

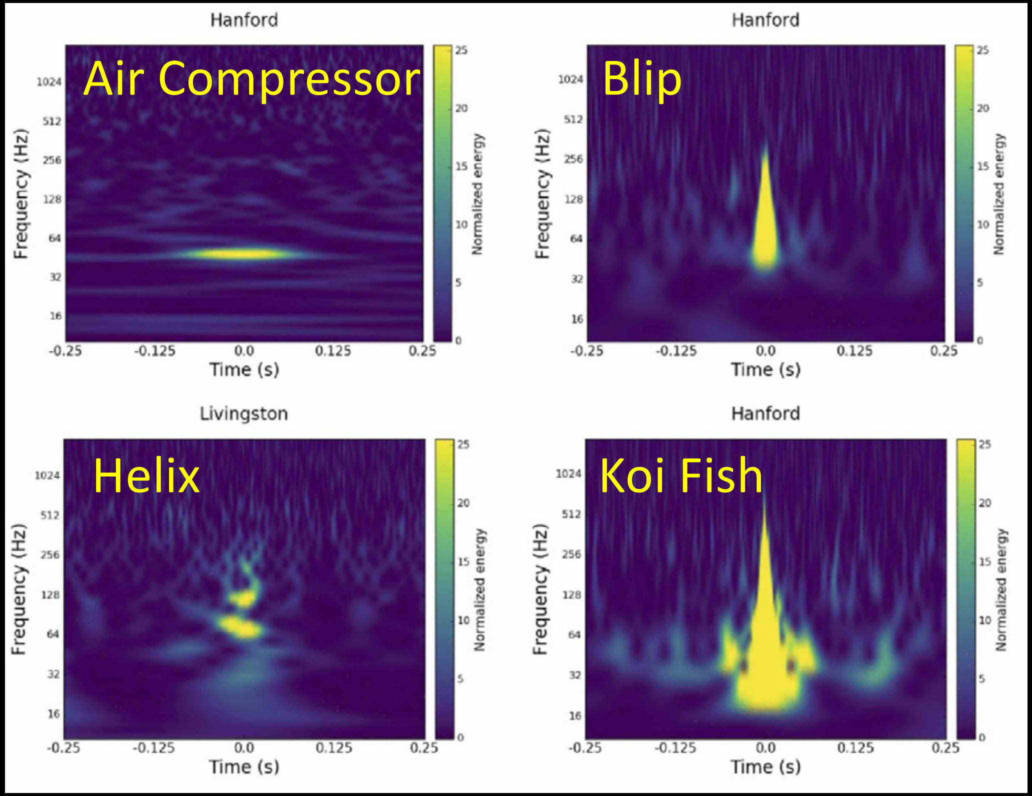
… but can’t tell where a signal is coming from.



GW190425, observed clearly only by LIGO-Livingston B. P. Abbott et al., ApJL892:L3 (2020)

Single-Detector Observations are vulnerable to Noise

LIGO, Virgo, and KAGRA data contain non-Gaussian background noise fluctuations: **``glitches''**

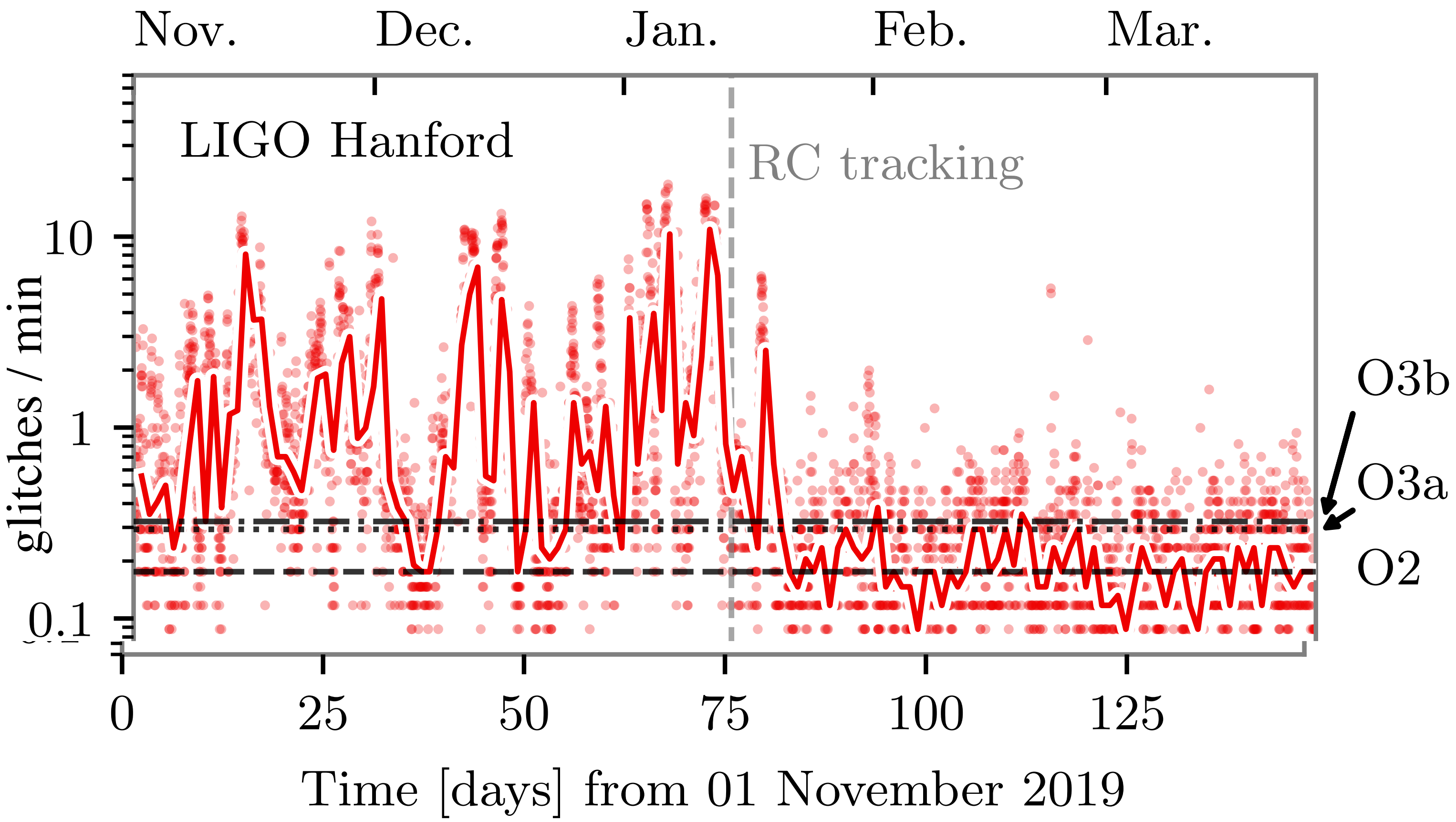


S. Bahaadini et al., Information Sciences 444 (2018) 172–186

The Hard Truth

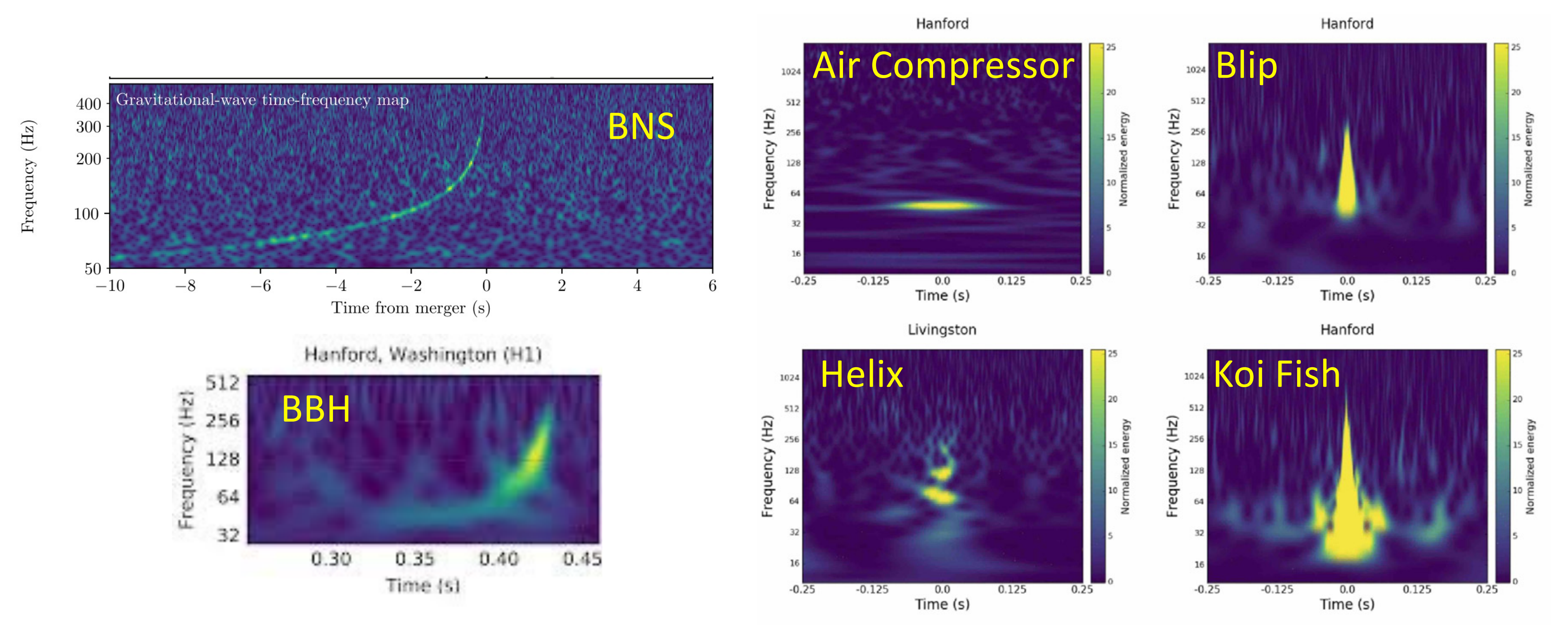
High-amplitude glitches (SNR>6.5) are very common.

- ➢ fatal for BBH and unmodelled burst searches.



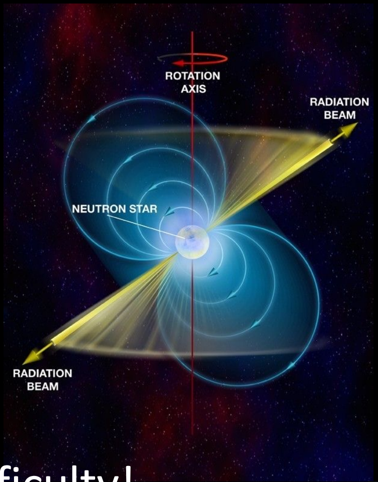
B. P. Abbott et al., 2111.03606

Only Complex Signals are Detectable by 1 IFO

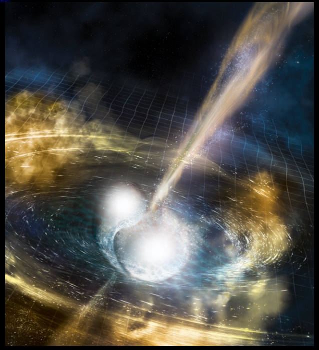


Single-Detector + Source

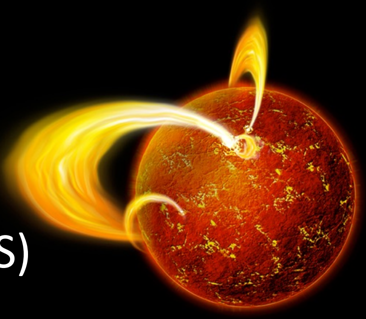
- Pulsars:
  - yes, “easy”



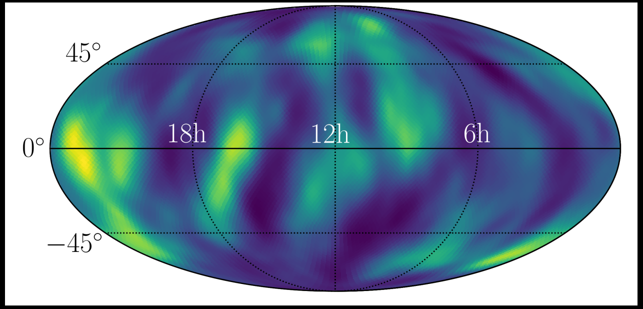
- CBCs:
  - BNS, with difficulty!
  - easier if unambiguous external counterpart (e.g. short GRB).
  - applies in principle to other [complex] well-modelled transients.



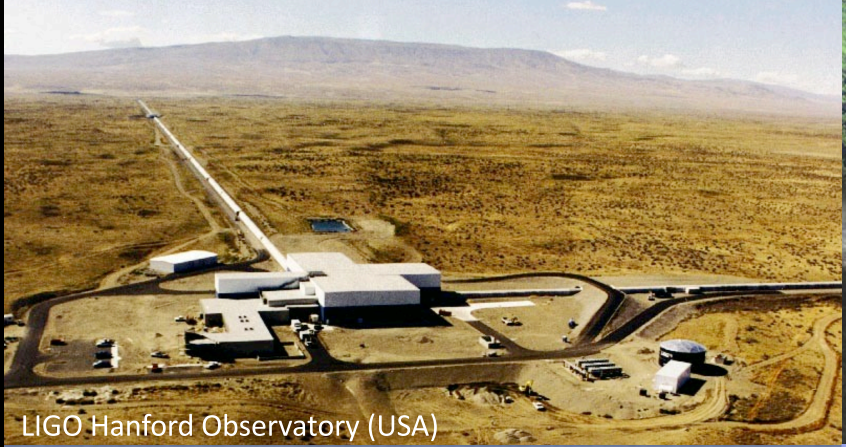
- Unmodelled Bursts:
  - theoretically possible with external trigger (e.g., EM flare, SNEWS) but *very difficult*



- Stochastic Backgrounds:
  - forget about them!



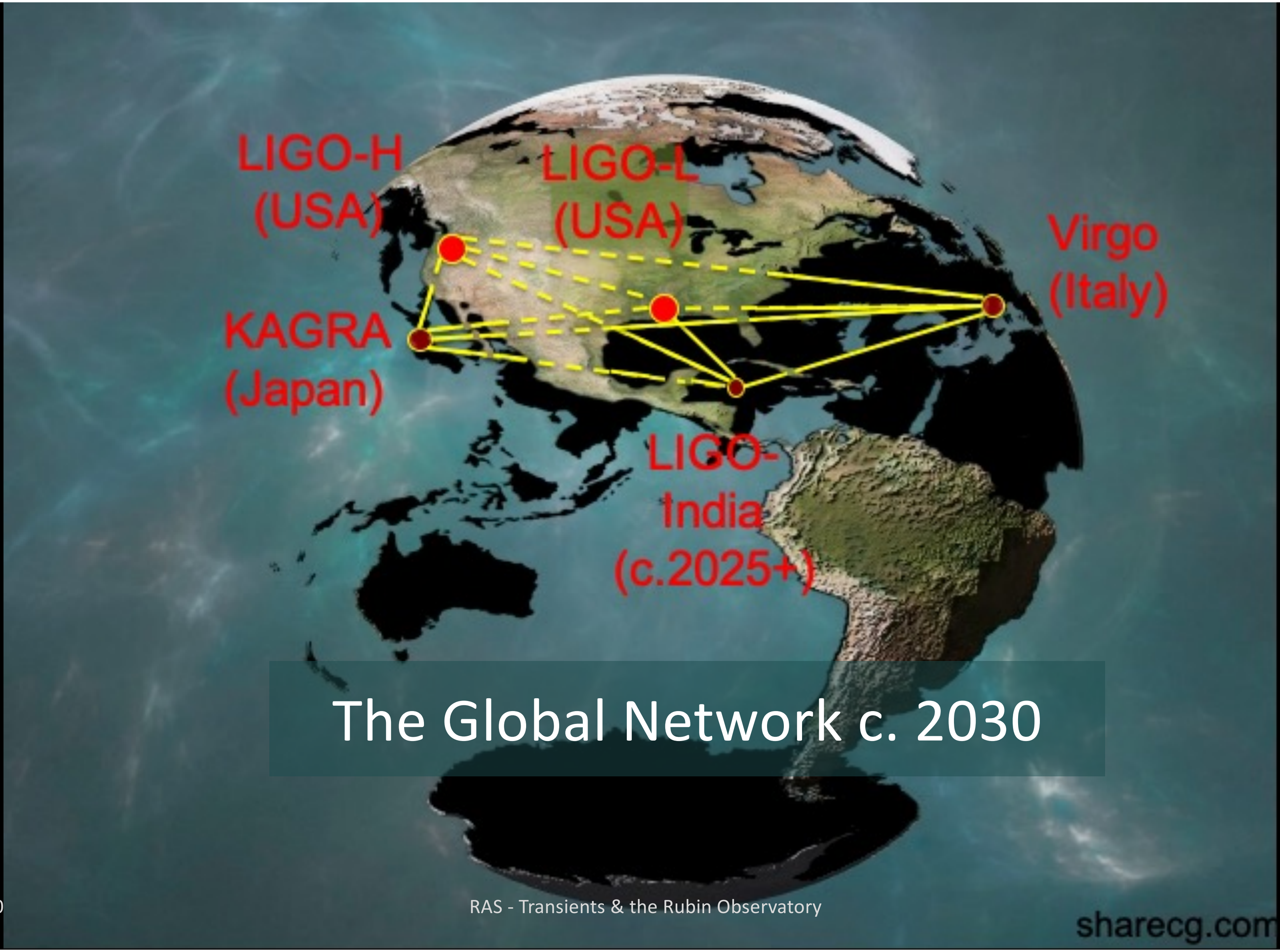
Multiple Detectors


LIGO Hanford Observatory (USA)

KAGRA Observatory (Japan)


LIGO Livingston Observatory (USA)


Virgo Observatory (Italy)



The Global Network c. 2030

The Hard Truth

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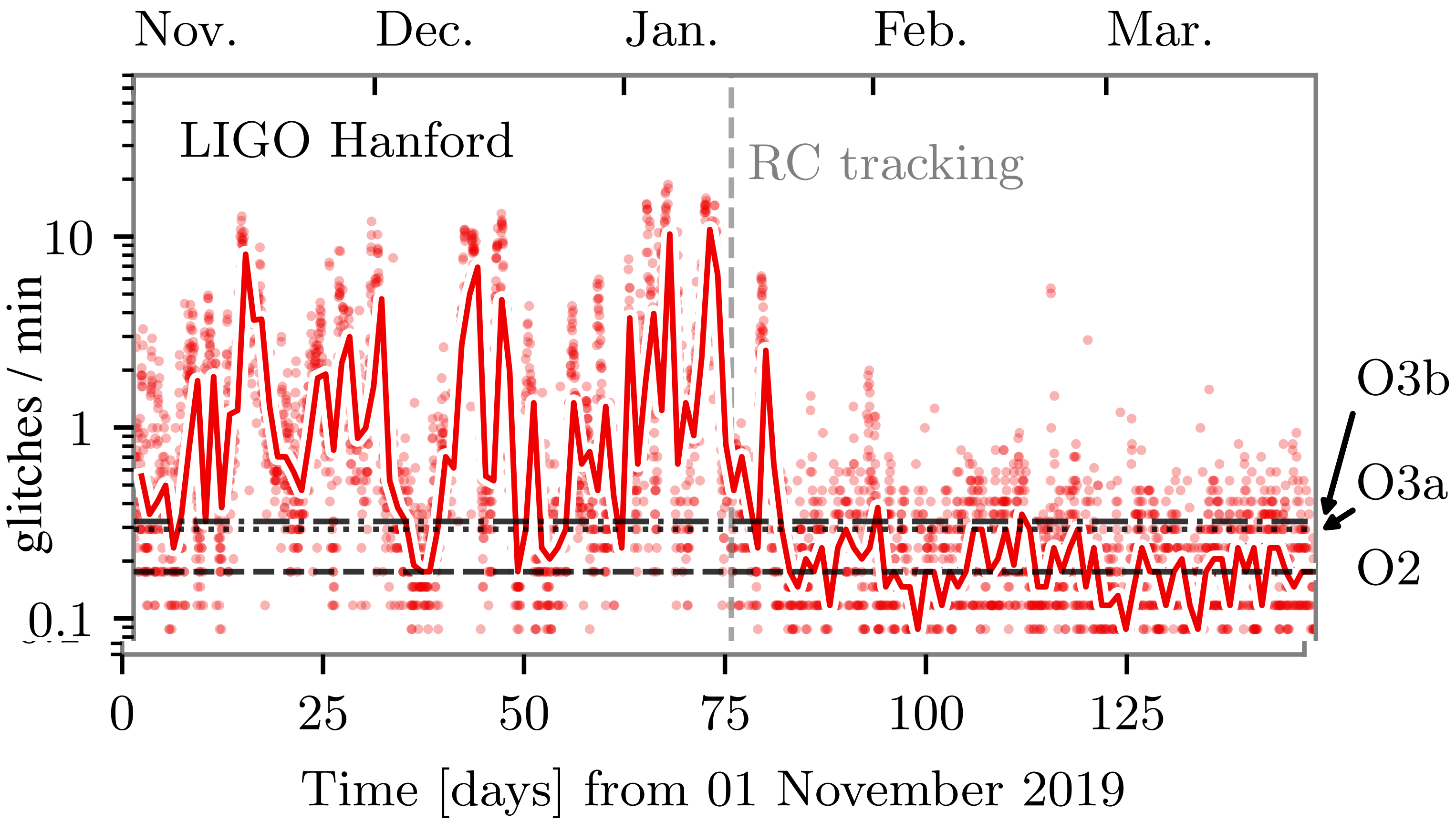


B. P. Abbott et al., 2111.03606

Detectors at 2+ sites are *vital*

- Drastic reduction of background in transient searches (CBCs, bursts):

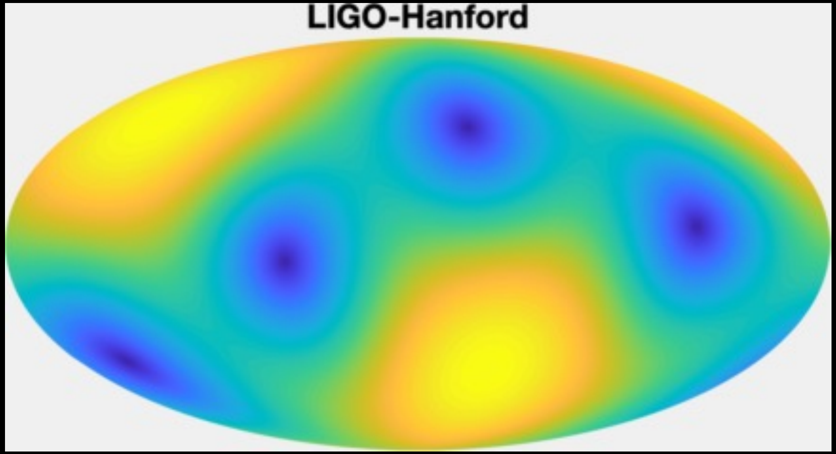
$$\mathrm{H}: \quad R_H = \frac{1}{60\,\mathrm{s}}$$

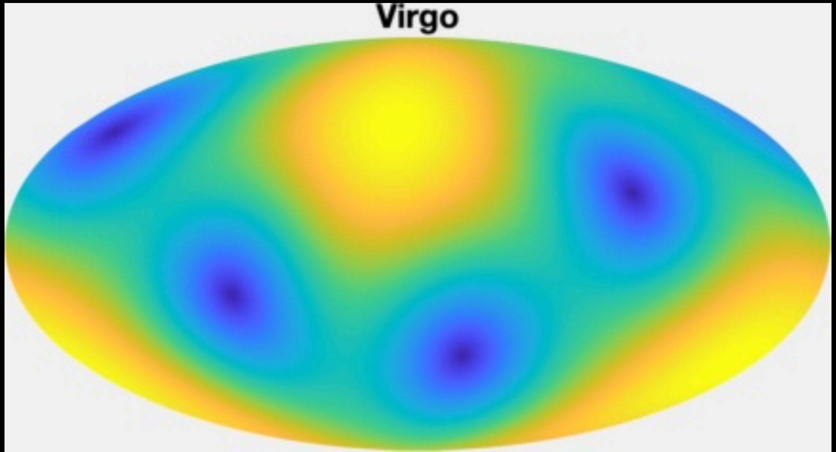
$$\mathrm{HL}: \quad R_H R_L (2\,T_{\mathrm{HL}}) = \left(\frac{1}{60\,\mathrm{s}}\right)^2 \times (2 \times 10\mathrm{ms}) = 0.5\,\mathrm{day}^{-1}$$

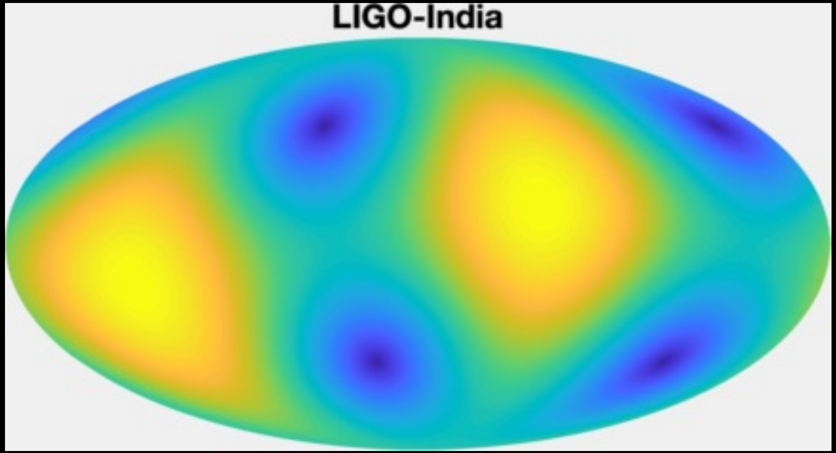
$$\mathrm{HLV}: \quad R_H R_L R_V (2\,T_{\mathrm{HL}})(2\,T_{\mathrm{HV}}) = \left(\frac{1}{60\,\mathrm{s}}\right)^3 \times (2 \times 10\mathrm{ms}) \times (2 \times 27\mathrm{ms}) = 0.2\,\mathrm{y}^{-1}$$

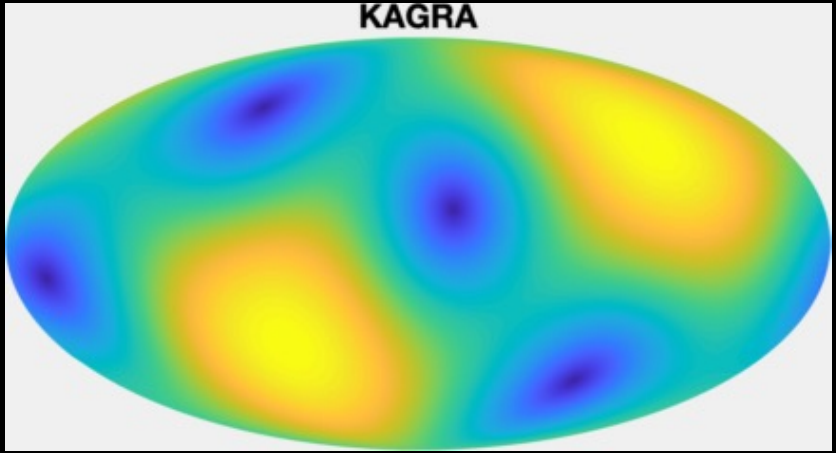
- Typical practice:
  - only require signal seen in at least 2 detectors
  - further reduce background rate by χ^2 matching against template (CBCs) or cross-correlation between detectors (bursts).
- Also: 2+ detectors required to make stochastic searches possible.

Network Sky Coverage

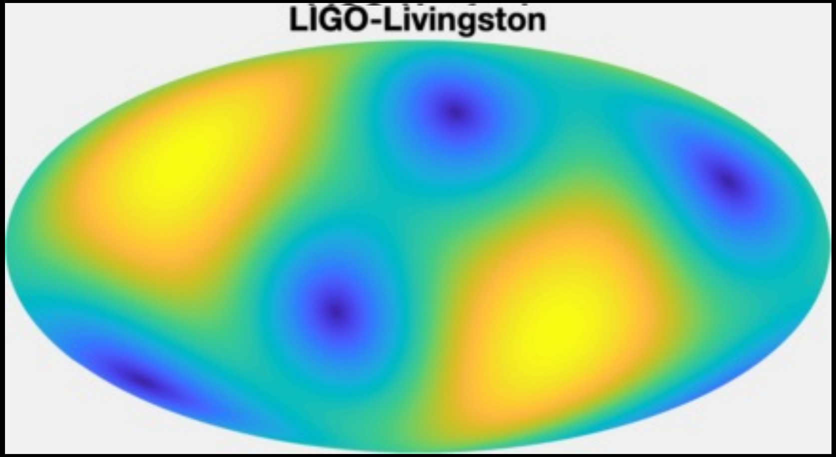

LIGO-Hanford

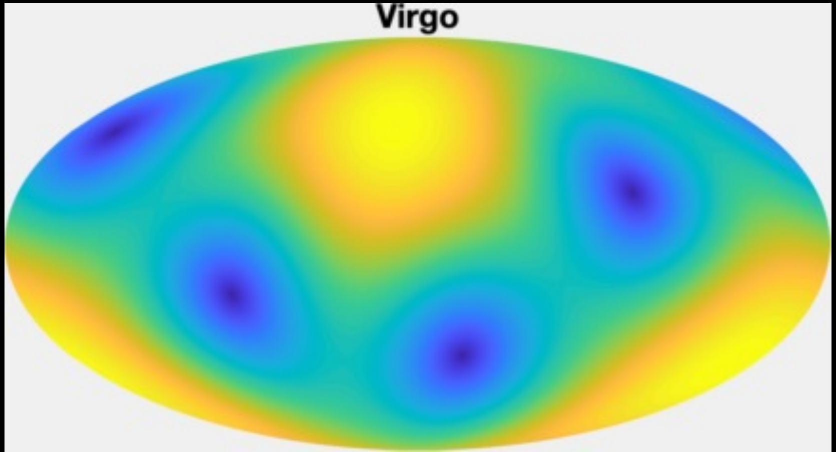

Virgo

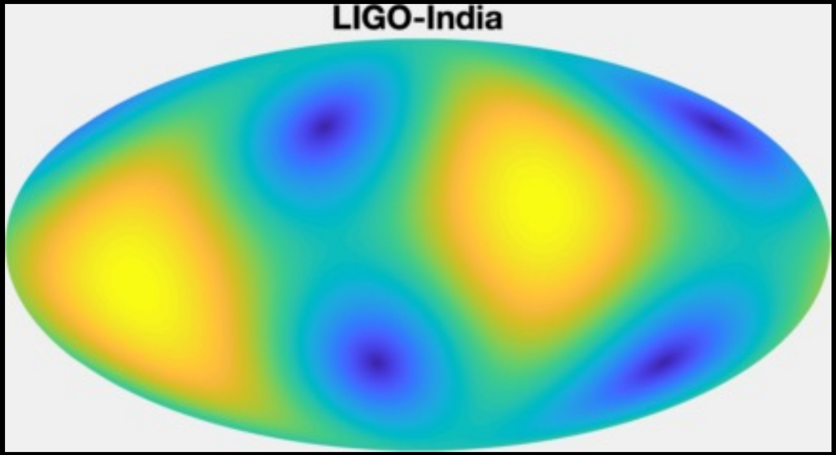

LIGO-India

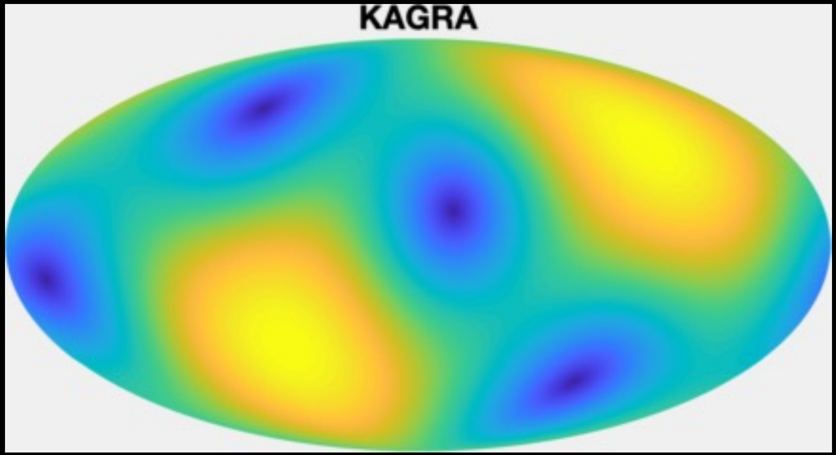

KAGRA

Network Sky Coverage





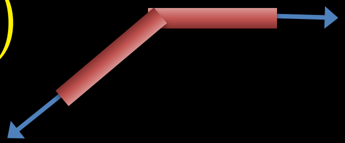




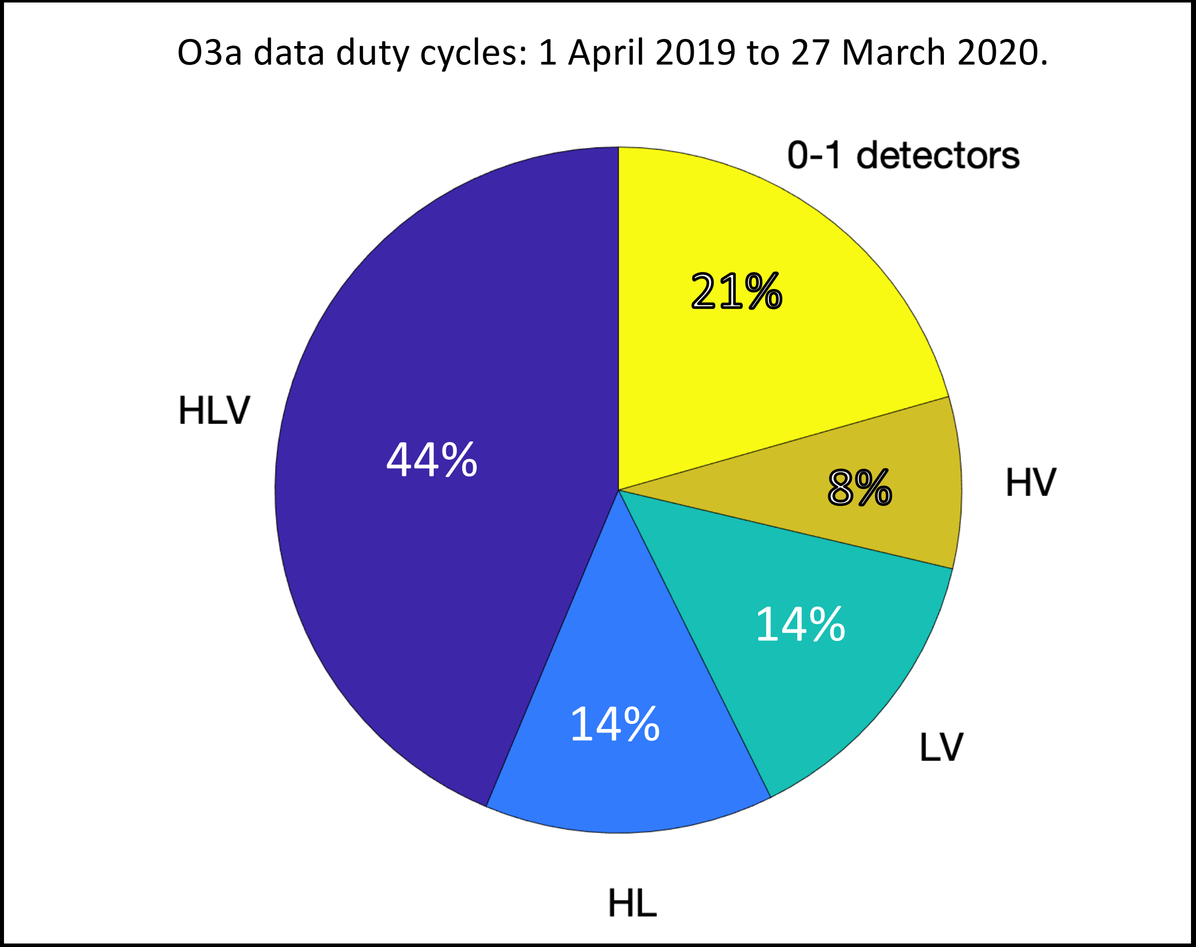
Network Overlap

	H	L	V	K	I
H	1.00	-0.89	-0.02	0.46	0.32
L		1.00	-0.25	-0.24	-0.55
V			1.00	-0.36	-0.15
K				1.00	0.21
I					1.00

$$O = 2 * \mathrm{Tr}(d_i^T d_j) \qquad d_i = 0.5(\hat{x}\hat{x}^T - \hat{y}\hat{y}^T)$$



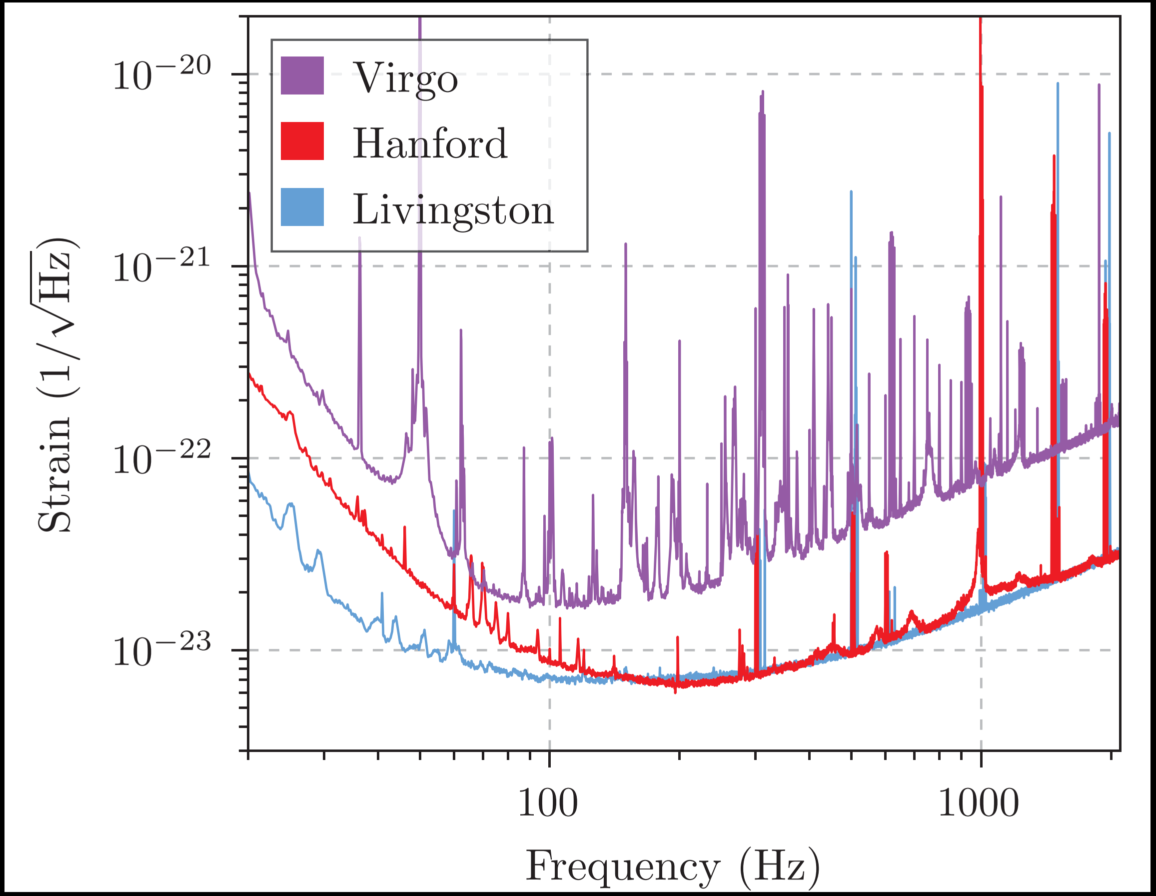
Coverage in Time



typical detector uptime:
70% - 75%

Network Signal-to-Noise Ratio (SNR)

B. P. Abbott et al., PRL119, 141101 (2017)

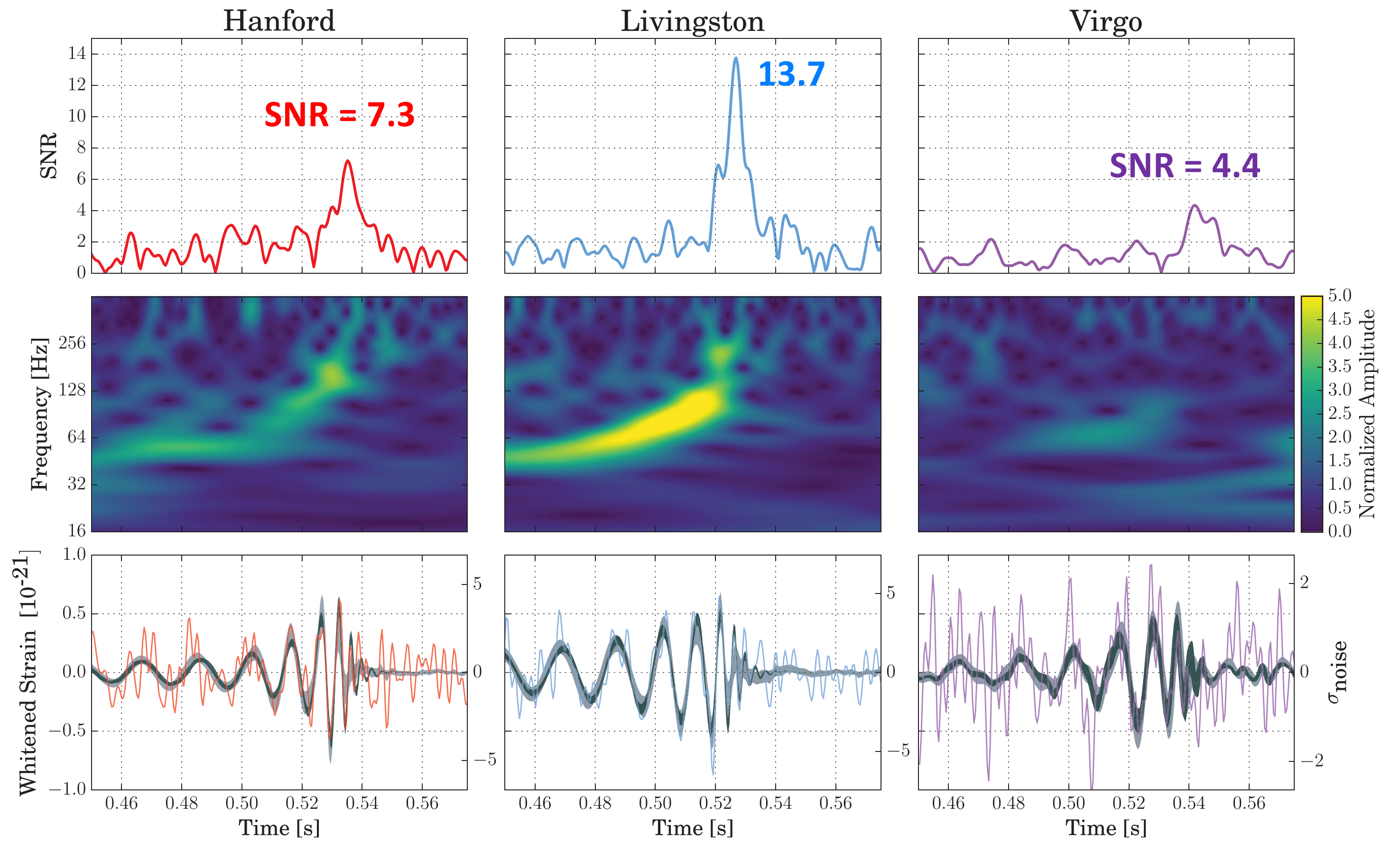


Combine data coherently for higher SNR.

- SNRs add in *quadrature*.

LIGO & Virgo at the time of GW170814

Network Signal-to-Noise Ratio (SNR)



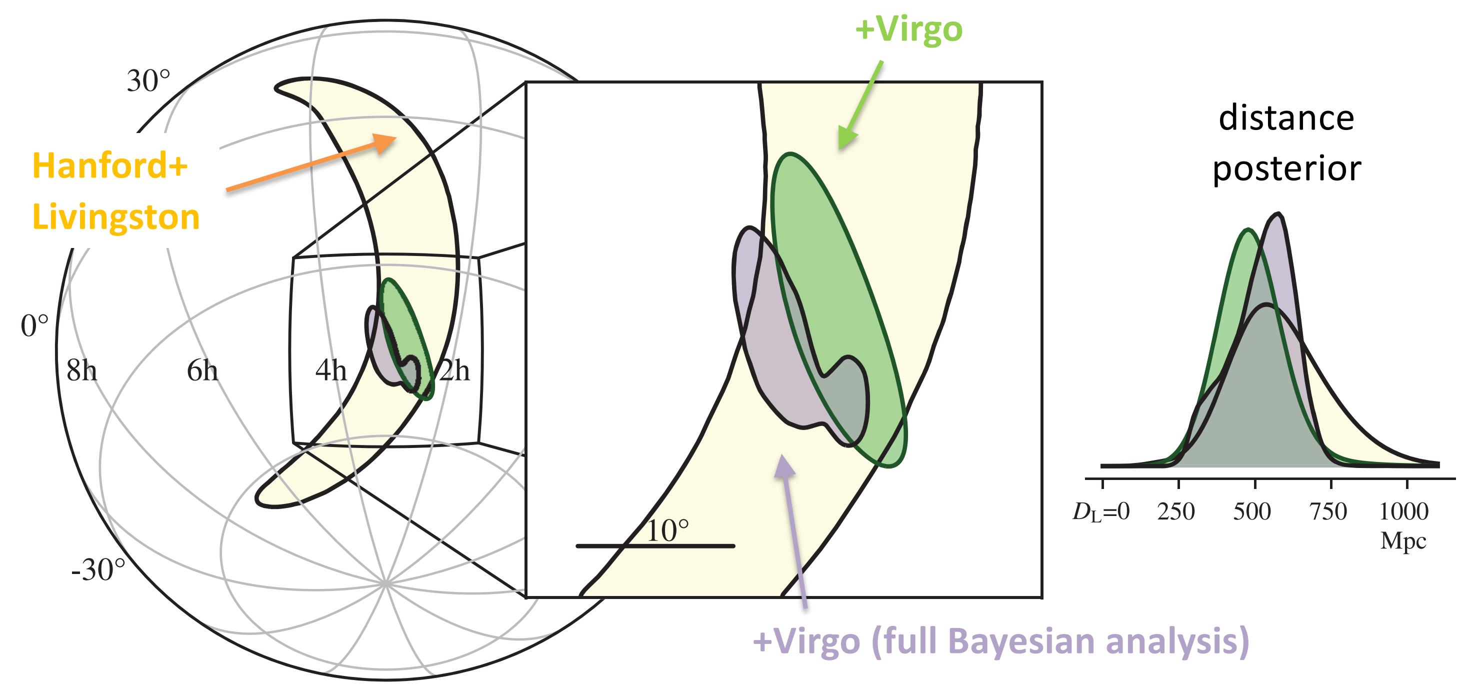
GW170814: Virgo's first detected GW

$$\frac{\sqrt{7.3^2 + 13.7^2 + 4.4^2}}{\sqrt{7.3^2 + 13.7^2}} = 1.04$$

- marginally more detections
- marginally better estimation of *intrinsic* source parameters (e.g. masses)

B. P. Abbott et al., PRL119, 141101 (2017)

Network Signal-to-Noise Ratio (SNR)



GW170814: Virgo's first detected GW

$$\frac{\sqrt{7.3^2 + 13.7^2 + 4.4^2}}{\sqrt{7.3^2 + 13.7^2}} = 1.04$$

- marginally more detections
- marginally better estimation of *intrinsic* source parameters (e.g. masses)
- dramatically better *extrinsic* parameters (sky location, inclination)

B. P. Abbott et al., PRL119, 141101 (2017)

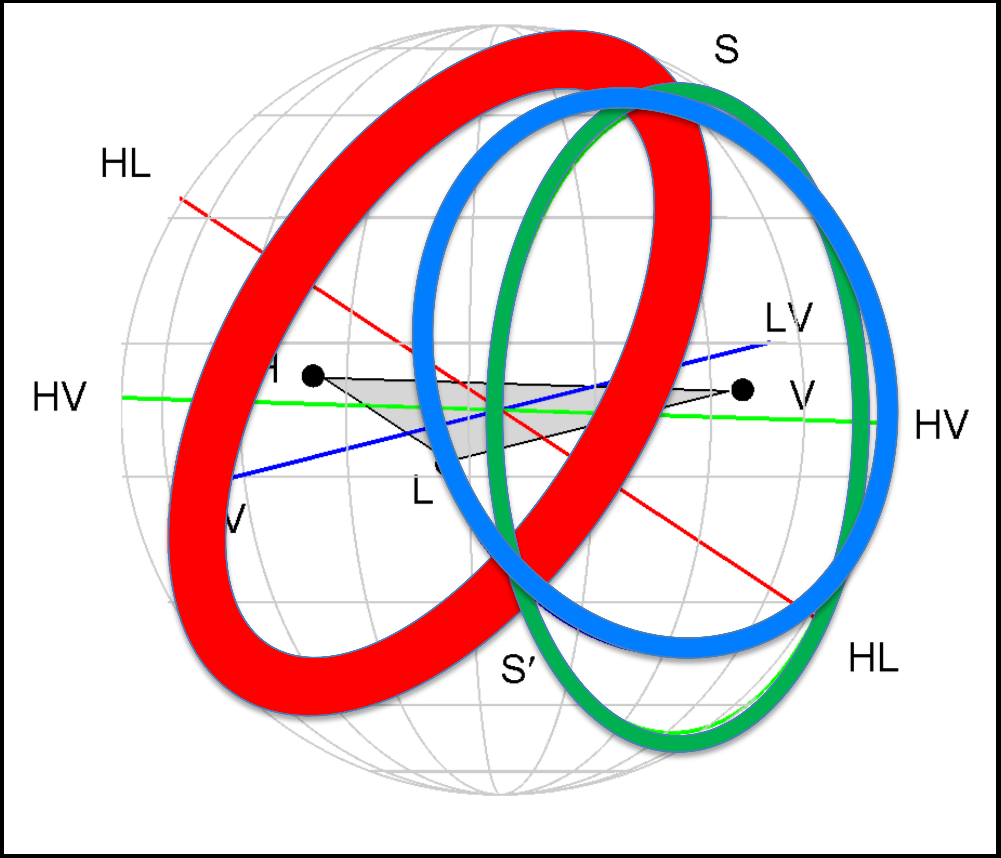
Sky Localisation: The Basics

- Localisation is primarily from timing-based triangulation.
  - plus corrections from amplitude, phasing
  - error boxes: intersecting annuli

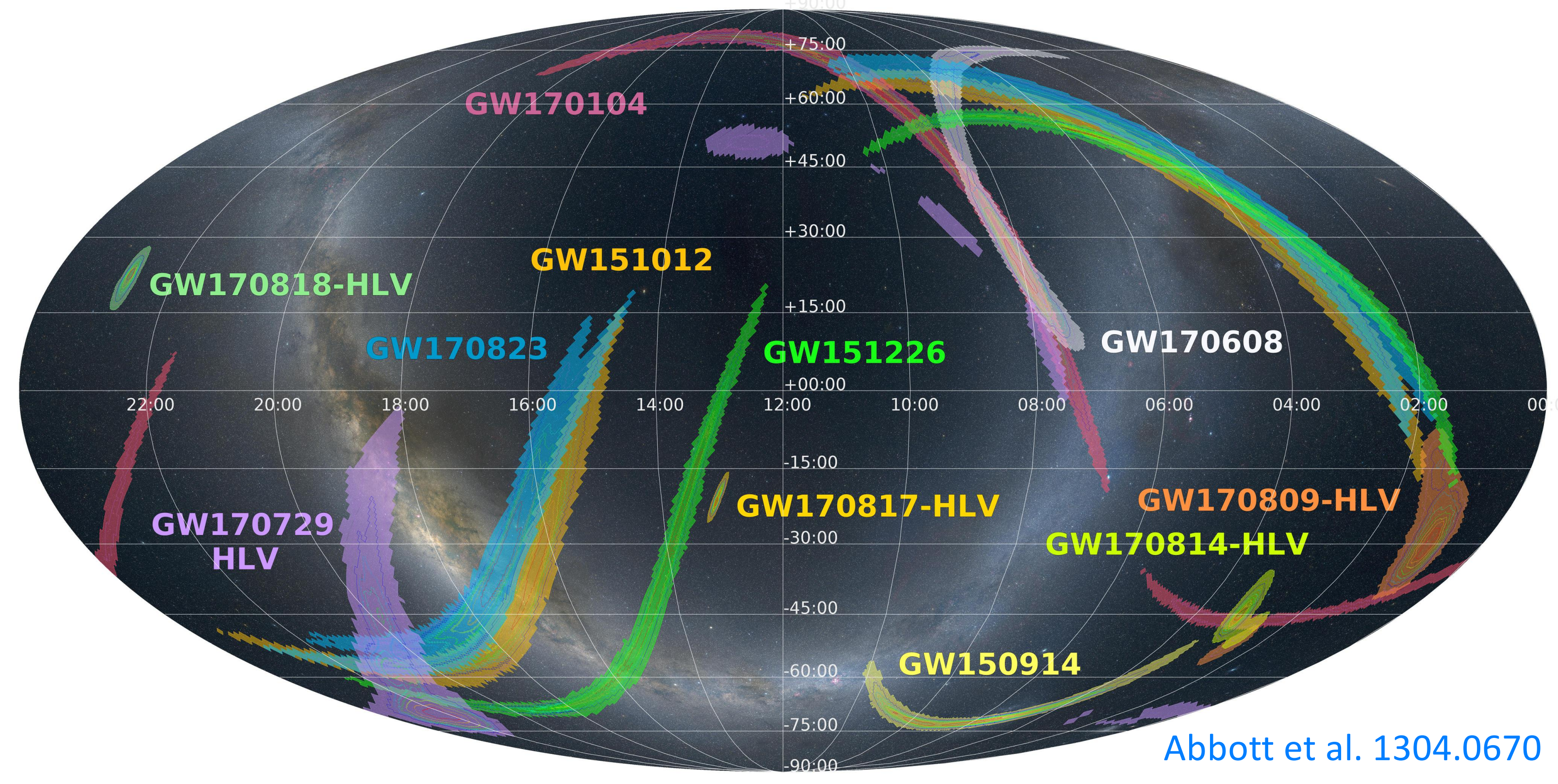
- For sources near threshold:

$$\Delta\theta \sim 1^\circ \left(\frac{100\,\mathrm{Hz}}{\Delta f}\right)\left(\frac{10\,\mathrm{ms}}{b/c}\right)\left(\frac{8}{\rho}\right)$$

Fairhurst 2010, Chatterji et al. 2006, Abbott et al. 1304.0670



Sky Localisation in O1, O2 (2015-17)

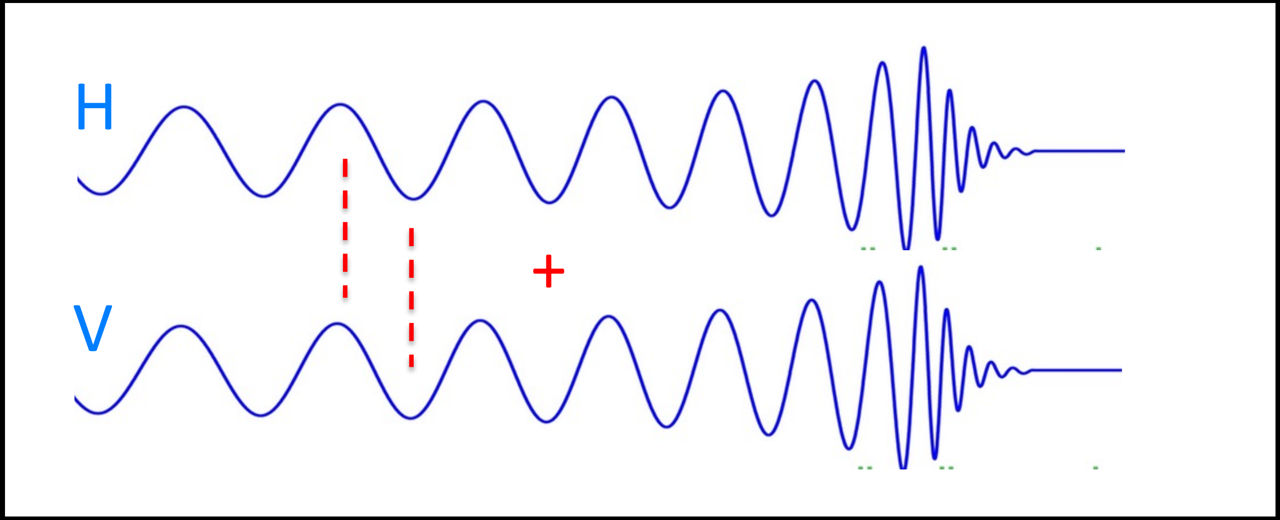


Abbott et al. 1304.0670

Refined analysis: 16 sq deg – 1666 sq deg (90%)
Initial alerts can be x1 – x2 larger areas.

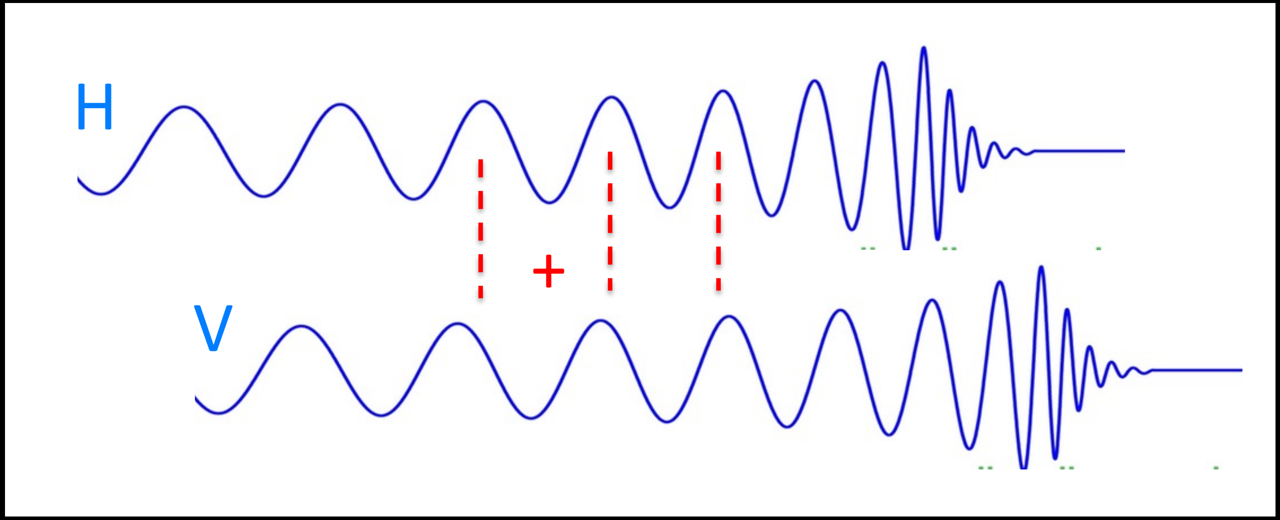
Signal Bandwidth & Multiple Rings

True source location:
use correct time delay to line up signal in each detector

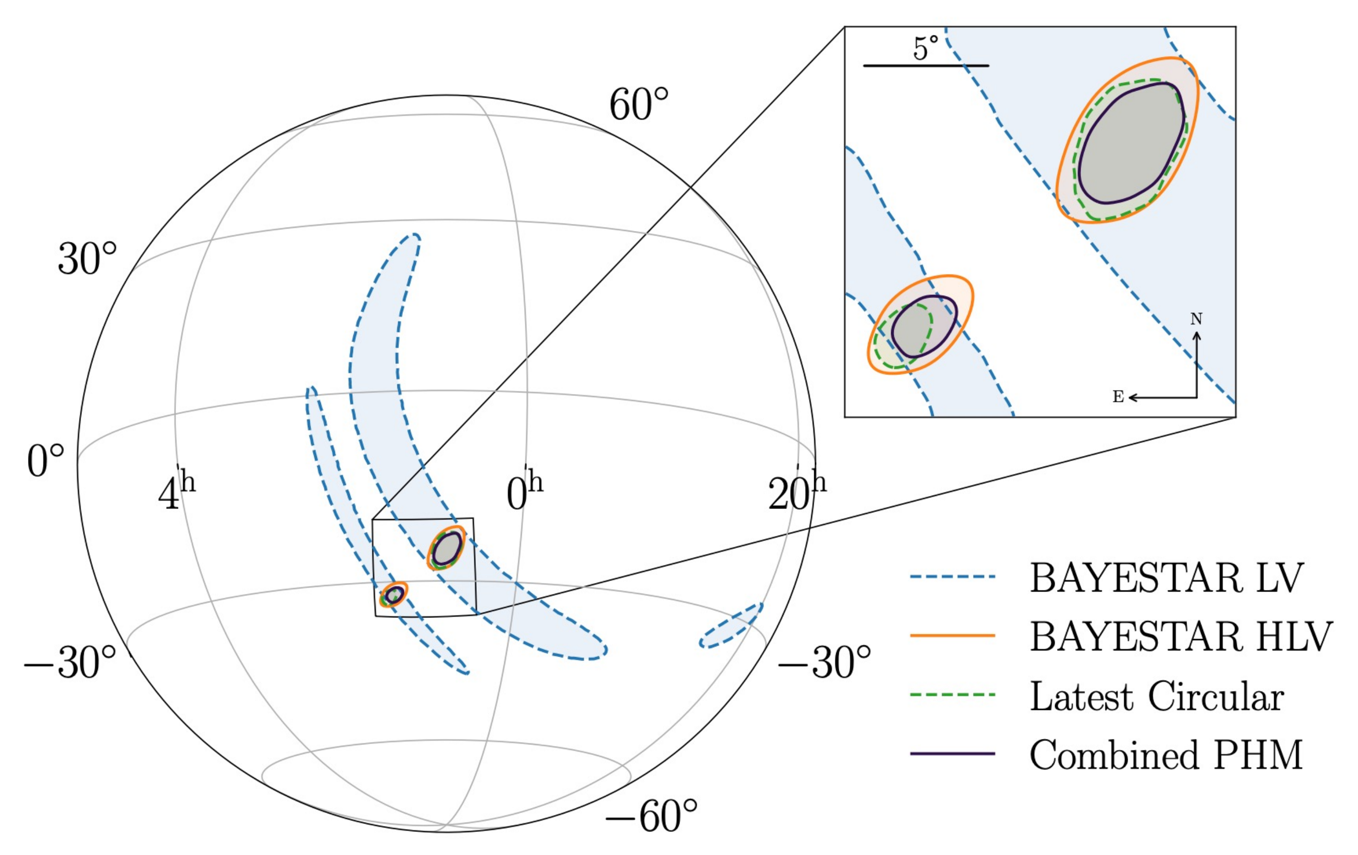


Signal Bandwidth & Multiple Rings

(Some) wrong locations:
time delay off by one cycle



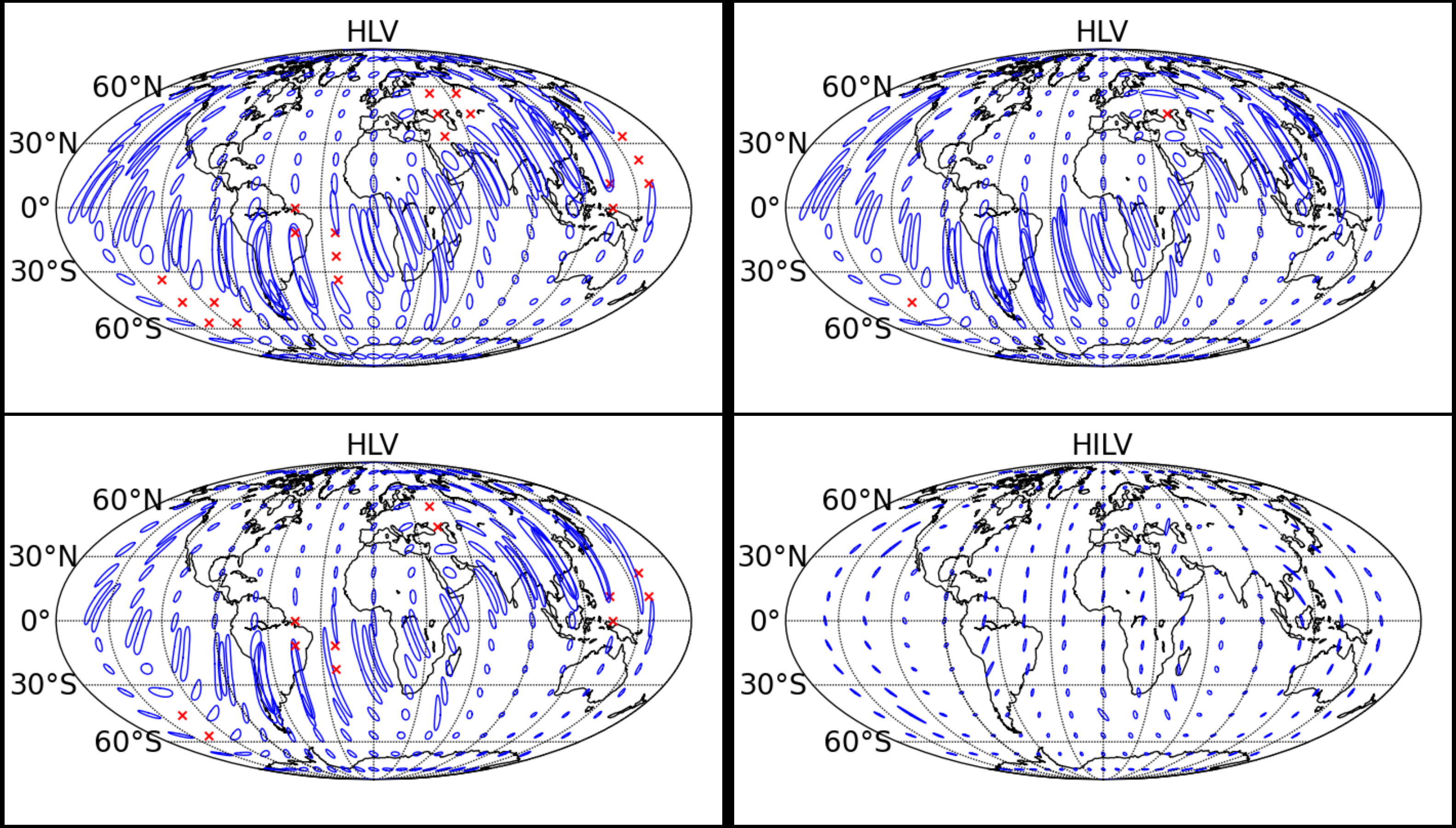
GW190814



5°
60°
30°
0°
4h
0h
20h
−30°
−30°
−60°
N
E
BAYESTAR LV
BAYESTAR HLV
Latest Circular
Combined PHM

A Look Ahead

B. P. Abbott et al. 1304.0670v1



Where We Are Now & What's Next

- GW emission & the quadrupole approximation
- Main source types and how we search for them
- Detector networks: detection confidence & sky localization
- Next: Results from LIGO & Virgo (Erik)

Leemage via Getty Images

and my favourite xkcd.com comic